not in G.L.

Peter Nicholson.

Drawn & Engraved by Edw.d Train.?

THE

GUIDE TO RAILWAY MASONRY,

CONTAINING

A COMPLETE TREATISE

ON

THE OBLIQUE ARCH.

In Four Parts.

PART I. CONTAINING ALL THE PRACTICAL GEOMETRY REQUISITE IN THE CONSTRUCTION OF ARCHES IN GENERAL.

PART II. THE FIRST PRINCIPLES OF DESCRIPTIVE GEOMETRY APPLIED TO THE THEORY OF OBLIQUE ARCHES; WITH PRACTICAL ILLUSTRATIONS.

PART III. THE PRINCIPLES OF CALCULATION ON THE VARIOUS LINES, ANGLES, AND PARTS OF OBLIQUE ARCHES.

PART IV. PRACTICAL CONSTRUCTION OF THE OBLIQUE ARCH WITH SPIRAL JOINTS.

WITH AN APPENDIX.

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Author of "The Architectural Dictionary," "The Builder and Workman's Director," "The Cabinet Maker's Guide," "The Carpenter's Assistant," "Principles of Architecture," "Treatise on Projection," "Practical Builder," &c.

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OBLIQUE ARCH WITH SPIRAL JOINTS.

PREFACE.

In this undertaking, the general reader is not supposed to be much acquainted with scientific researches, but to have some knowledge of descriptive geometry; the author, therefore, before he could venture to explain the principles of the oblique arch, and to reduce them to practice, has been induced to lay before his readers those problems which would afterwards be required, and also that they may always be ready for the use of the working mason or inspector, without being under the necessity of having recourse to other publications. In this treatise, the geometrical constructions and the calculations are quite independent of each other. Each system is complete of itself, so that the reader who is not acquainted with the principles of algebra may pass the formulæ, and proceed with the explanation of the figures and diagrams. But such as are able to understand the literal expressions, will soon find their utility and the great convenience and facility which they afford in the construction of the oblique arch; for in making working drawings of developments to the full size, so much room is required to lay them down, that it is often difficult to find a place which will contain them. The advantages, therefore, of calculation are obvious. In this treatise, every useful length, or distance, or angle of an oblique arch, has been found principally by common arithmetic, from the doctrine of similar triangles. The principles of calculation have been applied to three oblique arches which have been executed.

In the author's "Treatise on Stone-cutting," the construction of the oblique arch is given by a reverse process to that employed in the present work. It was there considered as necessary first to form the cylindric surface, from which the spiral surfaces of the beds might be more easily and more exactly obtained. Stone-cutters are, however, generally inclined to regulate the face from the bed; and, upon consideration, as the bed might be wrought by straight edges, the reverse process gives greater facility in the execution of the work; but whether we commence working the arch-stones with the soffit or the bed, the same templets would be required. Previously, however, to working the bed, it will be necessary to ascertain the angle of the twist. The forms of the templets are exhibited at No. 1, No. 2, No. 3, No. 4, (Plate XXV); and they are not shown by any other author who has written upon the oblique arch. Of these templets, No. 3 and No. 4, called arch-squares, are employed in squaring the arch-stones. No. 3 is used in forming the beds and soffits, namely, having wrought one bed by means of the winding rules, No. 5, No. 6, the position and form of the cylindric surface of the soffit may be ascertained by applying No. 3 in the same manner as a common square; and having finished one bed and the soffit, the position and form of the spiral surface of the remaining bed may also be ascertained by No. 3. The other arch-square, No. 4, is employed in forming the end of the stone, which is also a spiral surface. In the application of these arch-squares, the curved edge must rest upon the cylindric surface of the soffit, and thus the three faces of every arch-stone may be determined. Here it may be observed, that as the workman is not restricted, he may work the beds at one operation, of any length which he may find convenient. Every other means besides this of forming the arch-stones will be liable

to great inaccuracy. Though the instrument, first described in the author's "Treatise on Stone-cutting," is derived from a correct principle, it is difficult to keep it in a steady position upon the stone; yet this is the instrument which is used in other publications for squaring the stones.

It is very inconvenient to lay out the whole development of an oblique arch to the full size; but it is only necessary to find the development of half the intradosal-line; for by making a mould to the half thus found, the other half may be drawn by reversing the ends of the mould, and placing the curve on the other side of the line of subtense, and thus we shall have the entire curve. (See Plate XXXV.)

Considering the great expense of the tables which are necessary for making the calculations, and the number of accurate figures represented in the plates, it is hoped that this will not only be found a cheap, but a useful publication, to all who are desirous of acquiring a thorough knowledge of the principles of the oblique arch.

The improvements in this edition are considerable. Among other things, the method of executing an oblique arch with plane joints being seldom wanted, has been removed from the Introduction to the Appendix at the end of the work. To make room for these improvements, that portion of the work containing the history of the oblique arch has been omitted, as not necessary to the use of the workman, but being rather an incumbrance, and causing a confusion in the pages.

Instead of the theory of the oblique arch, which very few workmen are able to comprehend, an explanation is given of the

model, No. 38, exhibited in the Newcastle Polytechnic Exhibition, as being more useful and more interesting to the workman. Plate XXXIV exhibits the plan and corresponding elevation of the Bridge over the River Gaunless, the angle of obliquity being 26° 54′. This plate is placed opposite page 35, where it is explained.

In page 8 of this Preface it is said, that "the forms of the templets exhibited at No. 1, No. 2, No. 3, No. 4, (Plate XXV,) are not shown by any other author who has written upon the oblique arch;" but it would have been more correct to have said that these templets are the invention of the author, Mr. Peter Nicholson. He does not pretend to be the inventor of the oblique arch; but by templets shown at *Y, Z, Z,* and γ, Plate 17, explained in page 55, and the moulds shown at *D* and *G*, Plate 19, explained in page 57 of the "Treatise on the Art of Masonry and Stone-cutting," the common arch-stones of every oblique arch were executed from the year 1828 to April, 1836, by the rules shown in that treatise, without any other assistance. With what success the whole of the author's rules are being applied to the execution of various oblique bridges, will be seen by the Testimonials at the end of the present work.

A

TREATISE ON THE OBLIQUE ARCH.

Introduction.

SECTION I.—GEOMETRY.

THE meaning of a right angle, an acute angle, an obtuse angle, and of a right-angled triangle, an acute-angled triangle, and an obtuse-angled triangle, is supposed to be understood either from the reader's own practice or otherwise: more than this, however, is in general but very imperfectly known. As angles so very frequently require to be found under various circumstances, the following short discussion upon their properties, their construction, and the construction of right-angled triangles, will render the knowledge of them clear and familiar to the understanding of the workman, for whose use this Treatise is principally intended :—

ON THE NATURE OF ANGLES.

If from the point in which the two straight lines forming a right angle meet each other, the arc of a circle be described to meet each line, and if the arc be divided into ninety equal parts, and straight lines be drawn from the centre through each point of division, the right angle will be divided into ninety equal angles, each of which is called a *degree;* and if each degree be again divided into sixty equal parts, and straight lines be drawn to the centre, as before, each of these small angles is called a *minute.*

A number having a small zero or cypher placed over the right hand shoulder of the figure, or last figure, shows this number to be as many degrees as the figure or figures express, and an accent placed in the same manner over a number, shows this number to be as many minutes as the figure or figures express. Thus $36° \; 23'$ mean thirty-six degrees, twenty-three minutes.

The point at the meeting of two lines which contain an angle, is called *the vertex* or *point of the angle.*

Every angle is measured by the arc of a circle, described from the vertex between the sides containing the angle.

NAMES OF LINES IN THE CONSTRUCTION OF ANGLES.

A straight line joining the two ends of an arc, is called the chord of that arc, or simply the *chord.*

The distance between the ends of an arc, taken with a pair of compasses, is called the *extent* or *subtense* of the arc.

A straight line drawn from the centre to meet the chord of an arc perpendicularly, will divide the chord into two equal parts; and if the straight line be produced to meet the arc, the arc will also be divided into two equal parts.

Each half of the chord is called the sine of the half-arc to which it is opposite.

The line drawn from the centre, to meet the chord perpendicularly, is called the *co-sine of the half-arc.*

Hence the radius, the sine, and the co-sine, form a right-angled triangle, of which the sine is opposite to the angle subtended by the half-chord.

PRINCIPLE OF CONSTRUCTING ANGLES

The chord of 60° is equal to the radius:

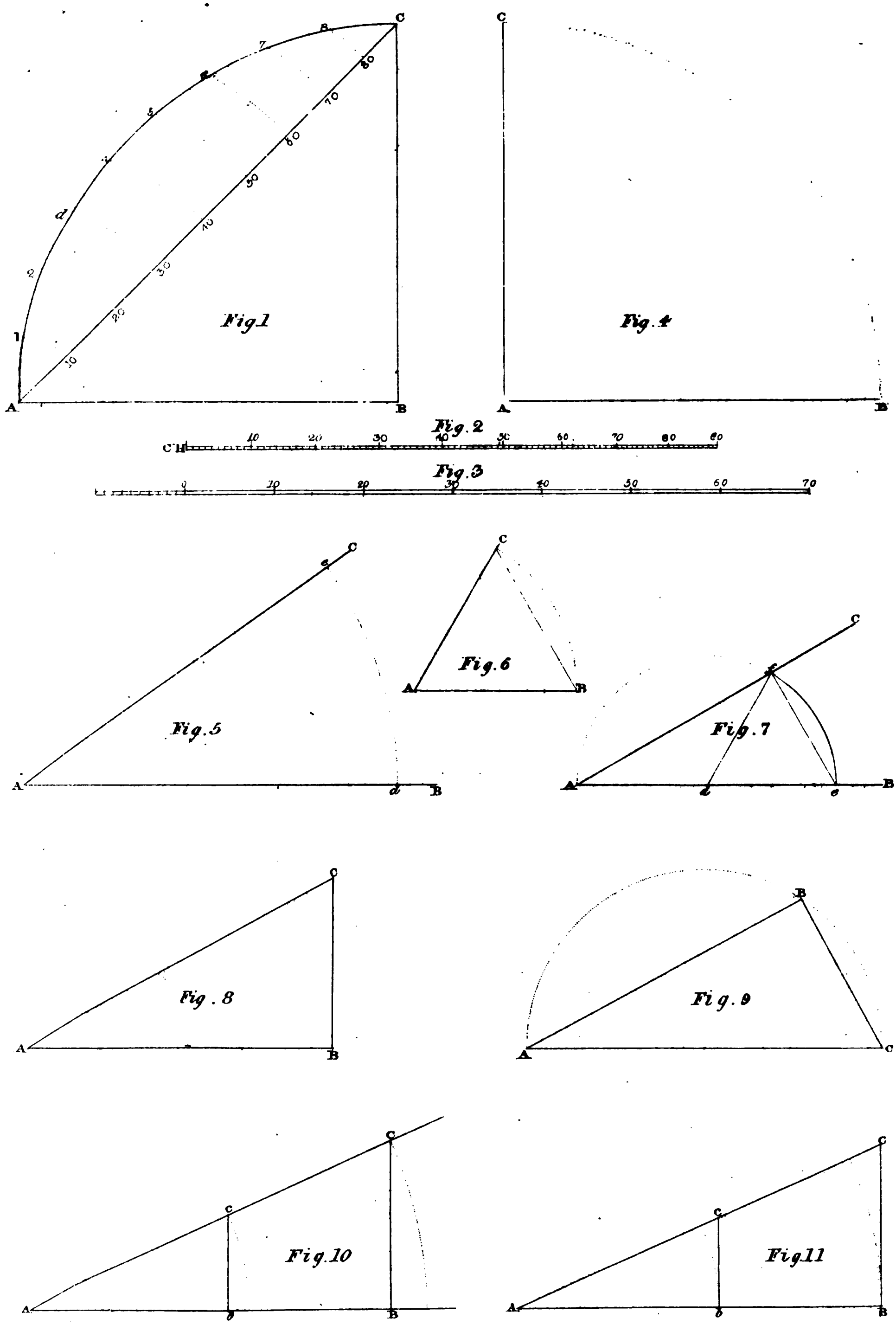
For upon any straight line as a diameter, describe a semi-circle. Divide the arc into three equal parts, and draw the two intermediate radii and the chords of the three arcs; then the figure will be divided into three isosceles triangles, of which each angle at the centre is one-third of two right angles; but if an isosceles triangle have the angle contained by the equal sides equal to one-third of two right angles, the triangle is equilateral; hence each of the two equal sides is equal to the third; but the two equal sides of each triangle are radii, and the third side is the chord of one of the equal arcs; therefore the radius is equal to the chord; but the angle at the centre being one-third of two right angles, is one-third of 180° equal to 60°; hence the chord of 60° is equal to the radius.

ON THE MEASUREMENT OF ANGLES.

The ratio of the radius and the chord of one arc being equal to the ratio of the radius and the chord of another arc, the angle contained by the two radii drawn from the extremities of the one arc, is equal

PLATE 1

Introduction



to the angle contained by the two radii drawn from the extremities of the other arc. Therefore if an arc be described with a radius equal to the chord of 60°, and if a given number of degrees taken from a scale of chords, and the extent placed upon the arc, and if through each extremity a line be drawn to the centre, the two radii will contain an angle of as many degrees, the radius being equal to the chord of 60°. If the extent set upon the arc be the chord of 90°, the angle contained by the two radii will be a right angle.

TO CONSTRUCT A SCALE OF CHORDS.

Draw a straight line *A B* (Fig. 1), and from *B* any point in *A B* draw *B C* perpendicular to *A B*. Make *B A* equal to the given radius, and from *B* with the distance *B A* describe the arc *A C*. From *C* with the distance *A B* cut the arc *A C* in *d*, and from *A* with the same radius cut the arc *A C* in *e*; then the quadrant *A C* is divided into three equal parts at the points *d* and *e*. Divide *A d*, *d e*, *e C*, each also into three equal parts, and the whole arc *A C* will thus be divided into nine equal parts. Divide each of these nine into ten equal parts, and the whole arc *A C* will be divided into ninety equal parts. Draw the chord *A C*, and from *A* with the distance of each point of division in the arc *A d e C*, cut the straight line *A C* as shown by the numbers 10, 20, 30, &c., and *A C* is a scale of chords.

Figure 2 is a scale of chords transferred from Figure 1, and is that which is referred to in the following constructions, and in the mensuration of angles.

Figure 3 is a scale of equal parts as used in the construction of triangles, and is the scale referred to. The parts may represent feet, yards, chains, &c.

CONSTRUCTING OF ANGLES.

From a given point *A* (Fig. 4), upon a given straight line *A B*, to construct a right angle by the scale of chords.

From *A*, with a chord of 60°, describe an arc *B C*; make *B C* equal to the chord of 90°; draw *A C*; and the angle *B A C* shall be a right angle, or *A C* shall be perpendicular to *A B*.

From a given point as a vertex, upon a given straight line, to construct an angle which shall contain a given number of degrees, &c.

From the vertex, with the subtense of 60° taken from the scale of chords, describe an arc meeting the given line; from the point of meeting, with a distance equal to the chord of the given angle, cut

the arc.; draw from the vertex a straight line through the point of intersection; and the two lines shall contain the angle required.

EXAMPLE.

At a given point *A* (Fig. 5), upon a given straight line *A B*, construct an angle of 35°.

From *A* with a chord of 60°, describe the arc *d e* meeting *A B* in *d;* from *d*, with the chord of 35°, cut the arc *d e* in *e;* through *e* draw *A C;* and the angle *B A C* shall contain 35°.

To make an angle equal to any number of degrees greater than 90°, but less than 180°.*

Let *A* be a given vertex, and *A B* one of the given lines containing the angle; from *A*, with the chord of 60°, describe an arc *B C;* from the point *B*, with the chord of 90°, cut the arc *B C* in *d;* make *d C* from the scale of chords equal to the excess of the given number of degrees above 90°; draw *A C;* and *B A C* is the angle required.

An angle *B A C* (Fig. 5), being given, to find the number of degrees it contains.

From *A*, with the chord of 60°, describe an arc *d e*, meeting *A B* in *d*, and *A C* in *e;* apply the distance *d e* to the scale of chords; and the point of extension will show the number of degrees. This will be found to be nearly 35°.

To make an angle *B A C* (Fig. 6), of 60°, without using the scale of chords.

From *A*, with any radius, describe the arc *B C;* from *B*, with the same radius, cut the arc *B C* in *C;* join *A C;* and *B A C* is the angle required.

For if a straight line *B C* be drawn, *A B C* will be an equilateral triangle, of which the angle *B A C* will be 60°, as well as the angles at *B* and *C*.

To make an angle *B A C* (Fig. 7), of 30°, without using the scale of chords.

Take from *A* upon *A B*, the two equal distances *A d, d e;* from *d* with the distance *d e* describe the arc *e f;* from *e* with the same distance cut the arc *e f* in *f;* through *f* draw *A C;* and *B A C* is the angle required.

For if the chord *e f* and the radius *d f* be drawn, *d e f* will be an equilateral triangle; and if the semi-circle *A f e* be completed, the

* The student is requested to supply the diagram for this proposition, it being omitted for want of room.

angle *e d f* at the centre will be double the angle *e A f* or *B A C* at the circumference; but *e d f* being an equilateral triangle, the angle *e d f* is 60°; therefore the angle *B A C* is 30°.

OF THE COMPLEMENT AND SUPPLEMENT OF AN ANGLE.

The difference between any angle and a right angle is called the complement of that angle.

Thus 55°, the difference between 35° and 90°, is called the complement of 35°; and reciprocally 35°, the difference between 55° and 90°, is the complement of 55°.

The difference between an angle and two right angles, is called the supplement of that angle.

Thus 143°, the difference between 37° and 180°, is called the supplement of 37°, and reciprocally 37°, the difference between 143° and 180°, is the supplement of 143°.

TRIANGLES.

The angles, as well as the sides of a triangle, are called *parts*, which are therefore six in number, of which any three being given, except the three angles, the other three may be found. In a right-angled triangle, as the right angle is always given, any two of the remaining parts, except the two angles, will be sufficient to construct the triangle; and thus in respect of the given data, we have five cases of right-angled triangles, which are as follows:—

Two parts being given to construct a right-angled triangle.

CASE I.

Given the two sides, which contain the right angle, to construct the triangle.

From the same point draw two straight lines perpendicular to each other; make these lines respectively equal to the two sides; join the unconnected extremities; and the figure will be the triangle required.

EXAMPLE.

Construct a right-angled triangle, of which one side shall be 35, and the other 19 feet.

Draw *A B* (Fig. 8), and *B C* perpendicular to *A B*; make *A B* equal to 35 feet, and *B C* equal to 19 feet; join *A C*; and *A B C* is the triangle required.

CASE II.

Given the hypothenuse and one of the other two sides, to construct the triangle.

Draw a straight line, and make it equal to the hypothenuse. Upon the hypothenuse, as a diameter, describe a semi-circle; from one end of the diameter, with the length of the other side, cut the semi-circular arc; join the point of intersection and each end of the diameter; and the figure is the triangle required.

EXAMPLE.

The hypothenuse being 40 feet, and the other side 35, construct the triangle.

Draw *A C* (Fig. 9); make *A C* equal to 40 feet; upon *A C*, as a diameter, describe the semi-circle *A B C;* from *A* with the length 35 feet of the other side, cut the semi-circular arc in *B;* join *B A*, *B C;* and the figure *A B C* is the triangle required.

CASE III.

Given one of the two sides about the right angle, and the angle adjacent to that side, to construct the triangle.

Draw a straight line; make it equal to the given length; from one end raise a perpendicular; at the other end of the given line make an angle equal to the given angle; and the figure made by the meeting of the lines shall be the triangle required.

EXAMPLE.

One side about the right angle being 35 feet, and the adjacent angle 28° 30′, construct the triangle.

Draw *A B* (Fig. 8); make *A B* equal to 35 feet; draw *B C* perpendicular to *A B;* make the angle *B A C* equal to 28° 30′; and the figure *A B C* is the triangle.

CASE IV.

Given one of the two sides about the right angle, and the opposite angle to that side, to construct the triangle.

Take the complement of the given angle, and we shall have an angle adjacent to the given side; therefore, proceed as in Case III, and the figure constructed shall be the triangle required.

EXAMPLE.

Given one side about the right angle equal to 35 feet, and the opposite angle equal to 61° 30′, construct the angle.

The complement of 61° 30′ is 28° 30′; proceed now as in the Example to Case III, and this shall be the triangle required.

CASE V.

Given the hypothenuse and an angle to construct the triangle.

Draw a straight line; make it equal to the hypothenuse; upon the given line as a diameter describe a semi-circle; make an angle with the diameter at one of its ends equal to the given angle; draw a straight line from the other extremity to the point of intersection; and the figure shall be the triangle required.

EXAMPLE.

Given the hypothenuse equal to 40 feet, and one of the angles 28° 30′, construct the triangle.

Draw *A C*, (Fig. 9); make *A C* equal to 40 feet; upon *A C* describe the semi-circular arc *A B C;* make the angle *C A B* equal to 28° 30′; draw *B C;* and the figure *A B C* is the triangle required.

In any of the five cases, to find the parts of the triangle required. If angles, they may be ascertained from the scale of chords; and if sides, from the scale of equal parts.

THE NAMES OF THE THREE SIDES OF A RIGHT-ANGLED TRIANGLE.

If the hypothenuse be called radius, the side opposite to either of the acute angles is called *the sine* of that angle, and the remaining side, which with the radius contains the angle, is called *the co-sine* of the angle. (*See* page 2.) Thus in the triangle *A B C* (Fig. 10), if *A C* be called radius, the side *B C* opposite the angle *A* is called the sine, and side *A B* the co-sine of the angle *A*.

If one of the sides about the right angle be called radius, the other side is called *the tangent* of the opposite angle, and the hypothenuse *the secant* of the same angle. Thus in the right-angled triangle *A B C* (Fig. 11), if *A B* be called the radius, *B C* is called the tangent of the angle *A*, and *A C* the secant of the same angle *A*.

Given the three parts of an oblique-angled triangle, to construct the triangle.

CASE I.

Given the three sides, of which any two is greater than the third, to construct the triangle.

Draw a straight line, make it equal in length to one of the sides; from one end of the line with the length of another side describe an arc; from the other end describe another arc to intersect the other; join the point of intersection, and each end of the line; and the figure shall be the triangle required.

EXAMPLE.

Given the three sides respectively equal to 78, 57, and 38 feet, construct the triangle.

Draw *A B* (Fig. 1); make *A B* equal to 78 feet; from *A* with the distance of 57 feet, describe an arc; from *B* with the distance of 38 feet, describe another arc intersecting the former at *C;* join *C A, C B;* and the figure *A B C* is the triangle required.

CASE II.

Given the two sides and the contained angle, to construct the triangle.

Draw a straight line; make the line equal in length to one of the given sides; from one end of the line make an angle equal to the given angle; from the vertex upon the unlimited line, set off the length of one of the other two sides; join the unconnected ends of the two lines; and the figure shall be the triangle required.

EXAMPLE.

Given the two sides respectively equal to 78 and 57 feet, and the contained angle 27° 30′, to construct the triangle.

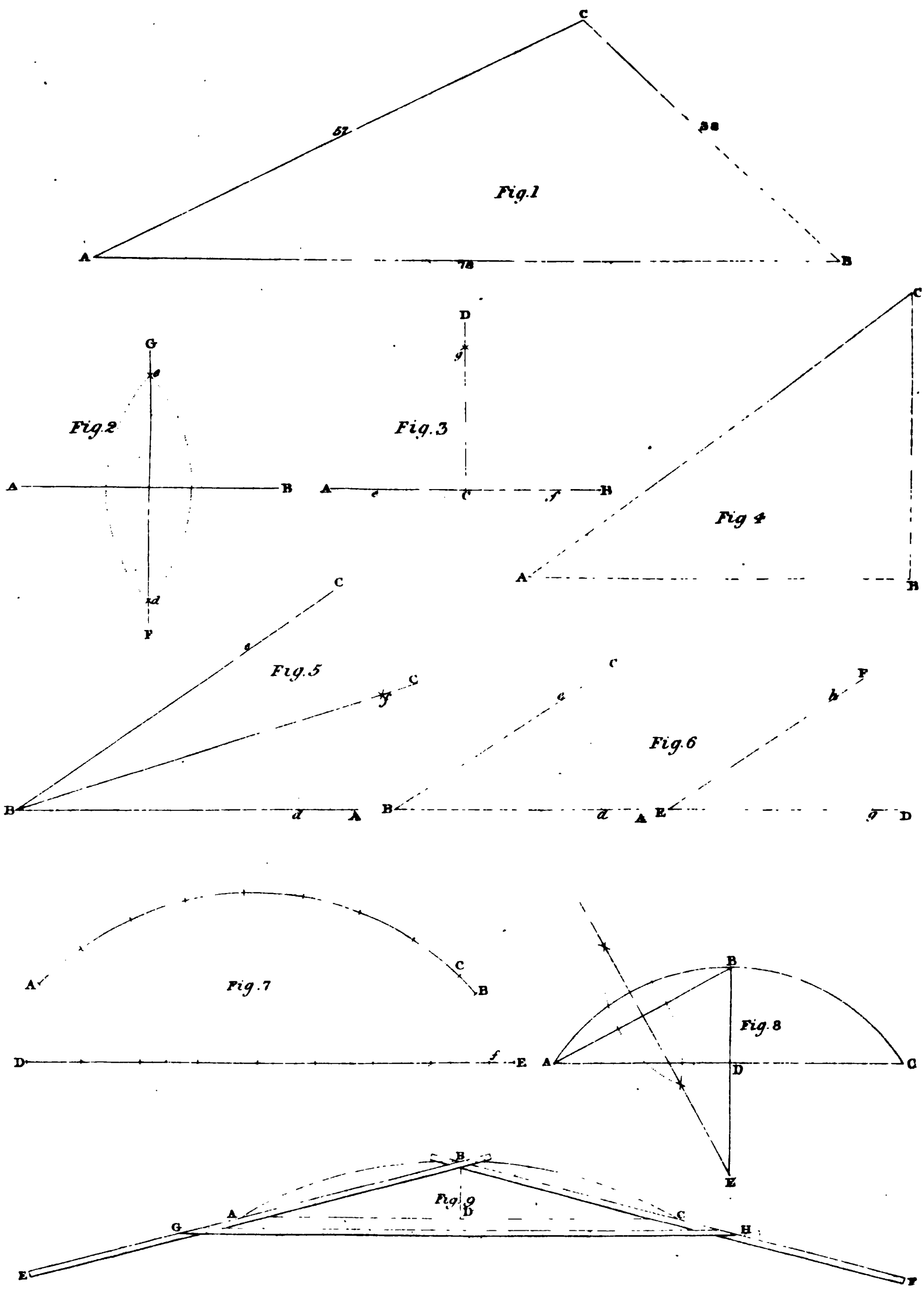
Draw *A B* (Fig. 1); make *A B* equal to 78 feet; make the angle *B A C* equal to 27° 30′; make *A C* equal to 57 feet; join *B C;* and *A B C* is the triangle required.

CASE III.

Given two sides, and an angle opposite to one of them, to construct the triangle.

PLATE 2.

Introduction



Draw a straight line; make the length of the line equal to one of the given sides; at one end of the line make an angle equal to the given angle; from the other end, with the length of the other side, cut the unlimited side of the given angle; join the end of the line and the point of intersection; and the figure shall be the triangle required.

EXAMPLE.

Given two sides of a triangle respectively equal to 78 and 57 feet, and the angle opposite to the side 57 equal to 43° 30′, to construct the triangle.

Draw *A B*, (Fig. 1); make *A B* equal to 78 feet; make the angle *A B C* equal to 43° 30′; from *A*, with the distance of 57 feet, cut *B C* in *C*; join *A C*; and *A B C* is the triangle required.

CASE IV.

Given a side, and the two adjacent angles, to construct the triangle.

Draw a straight line; make the line equal to the length of the given side; at each end of the line make angles respectively equal to the two given angles; and the figure shall be the triangle required.

EXAMPLE.

Given one side equal to 78 feet, and the two angles respectively equal to 26°, and 43° 30′, construct the triangle.

Draw *A B*, (Fig. 1); make *A B* equal to 78 feet; make the angle *B A C* equal to 27° 30′; the angle *A B C* equal to 43° 30′; and the figure *A B C* is the triangle required.

CASE V.

Given two angles and a side opposite to one of them, to construct the triangle.

In this construction, the line which is to be opposite to one of the given angles will be adjacent to the other.

Draw a straight line; make the line equal to the length of the given side; make an angle at one end of the line equal to the adjacent angle; make an angle at the other end of the line equal to the supplement of the sum of the two given angles; and the figure enclosed by the three lines is the triangle required.

EXAMPLE.

Given the two angles respectively equal to 27° 30′, and the side opposite to 43° 30′ equal to 57 feet, construct the triangle.

Draw *A C* (Fig. 1); make *A C* equal to 57 feet; make the angle *C A B* equal to 27° 30′; make the angle *A C B* equal to 109°, which is the supplement of 71°, the sum of 27° 30′, and 43° 30′; and the figure enclosed by the three straight lines is the triangle required.

In any case when a triangle is constructed to find the remaining parts, the length of the sides will be found by applying the lengths found by construction upon the scale of equal parts, and the measurement of angles by the scale of chords.

BISECTION OF A LINE PERPENDICULARLY.

To bisect a given straight line *A B* (Fig 2), that is, to divide it into two equal parts.

From *A*, with any distance greater than the half of *A B*, describe an arc; from *B*, with the same radius, describe another arc intersecting the former arc at *d* and *e*; through the points *d*, *e*, draw the straight line *F G*; and *F G* will bisect *A B* perpendicularly in *C*.

PROBLEMS REQUIRED IN PRACTICE.

PROB. I.—From a given point *C* (Fig. 3), near the middle of a straight line *A B*, to draw a perpendicular.

On each side of the point *C*, upon the line *A B*, take two equal distances *C e*, *C f*; with any radius greater than *C e* or *C f* describe an arc from the point *e*; with the same radius describe another arc from the point *f*, intersecting the former arc in *g*; through *g* draw the straight line *C D*; and *C D* is perpendicular to *A B*.

PROB. II.—From a given point *B*, (Fig. 4), at the end of a given straight line *A B*, to draw a perpendicular.

Make *A B*, taken from a scale of equal parts, equal to 4 feet; from *B*, with a distance equal to 3 feet, describe an arc; from *A*, with a distance equal to 5 feet, describe another arc intersecting the former in *C*; join *B C*; and *B C* is perpendicular to *A B*.*

* For if *A C* be drawn, and if the square of *A C* be equal to the sum of the squares of *A B*, *B C*, the triangle *A B C* shall be a right angled triangle (Euclid, Book I, Proposition 48), the right angle being at *B*; therefore, because $(n^2+1)^2 = (n^2-1)^2 + (2n)^2$, the numbers n^2+1, n^2-1, $2n$, are the sides of a right angled triangle. Thus, let $n=2$, then $n^2+1=5$, $n^2-1=3$, $2n=4$; therefore, because the

The same thing may be done by using the numbers 6, 8, 10, instead of 3, 4, 5, or any numbers in the proportion of 3, 4, 5.

PROB. III.—To bisect a given angle *A B C* (Fig 5).

From *B*, with any radius, describe an arc *d e*, meeting *B A* in *d*, and *B C* in *e;* from *d*, with any radius greater than half the distance between *d* and *e*, describe an arc; from *e* with the same distance, describe another arc meeting the former in *f;* through *f* draw *B C;* and *B C* will bisect or divide the angle *A B C* into two equal angles.

PROB. IV.—To make an angle at the point *E* (Fig. 6), with the straight line *D E*, that shall contain an angle equal to the given angle *A B C*.

From *B*, with any convenient radius, describe the arc *d e*, meeting *B A* in *d*, and *B C* in *e;* from *E*, with the same radius, describe an arc *g h*, meeting *E D* in *g;* make *g h* equal to *d e;* through *h* draw the straight line *E F;* and *D E F* is the angle required.

PROB. V.—To make a straight line equal to the length of a given arc.

The thing here proposed to be done cannot be effected by any rule founded upon geometrical principles. It is evident that if a given arc be divided into equal parts, and the chords of these arcs be as often repeated upon a straight line, the multiple thus extended will be less than the length of the arc; the smaller, however, the distance is between the points, and the more numerous the parts are, the straight line which contains the multiple shall be the more nearly equal to the length of the arc.

EXAMPLE.

Find the length of the arc *A B*, (Fig. 7).

Take a small distance between the points of the compasses, and suppose the arc to contain this distance eight times from *A* to *c* with a remainder *c B;* draw the straight line *D E*, upon which repeat the chord as often as the number of small equal arcs are contained upon the whole arc from *D* to *f;* make *f E* equal to the remaining part *c B* of the arc; then will *D E* be nearly equal to the arc *A B*.

sides of the triangle *A B C* are 5, 4, 3, it is right angled. The numbers 3, 4, 5, or their doubles, 6, 8, 10, are very convenient for the use of workmen. There are numbers, however, in a very different proportion that will answer the same purpose: thus, let $n=4$, then $n^2+1=17$, $n^2-1=15$, $2\,n=8$; hence 17, 15, 8, are the sides of a right angled triangle, but more inconvenient than the others.

OBSERVATION.

In practical works, when the points through which a curve required to be drawn are given by being previously found, the curve is drawn by placing small nails in the points, and bending a thin slip of wood, or other elastic substance, round the points, and by drawing a curve by the side of the slip next to the nails; then the length of the slip extended in a straight line will be equal very nearly to the length of the arc or curve. The length of the curve obtained in this way is perhaps as exact as it is possible to be found.

PROB. VI.—Upon a straight line *A C* (Fig. 8), as a chord, to describe the arc of a circle, the height of the middle of the arc being given.

Bisect *A C* by a perpendicular *B E* intersecting *A C* in *D;* make *D E* equal to the height of the arc; draw the chord *A B* of the half arc; bisect *A B* by a perpendicular, meeting *B E* in *E;* from *E,* with the distance *E B,* describe the arc *A B C;* and *A B C* is the arc required.

It frequently happens that the distance of the centre of the circle is so very great as to make it inconvenient to get a radius and the portion of the arc of the circle required so very small; recourse must be had to the method of describing an arc of a circle without using the centre, by means of an angle.

PROB. VII.—The same things being given to describe the arc, without making use of the centre.

Draw the straight line *A C* (Fig. 9), and make *A C* equal to the chord, bisect *A C* by a perpendicular *B D,* meeting *A C* in *D;* make *D B* equal to the height of the arc. Having prepared two thin slips of wood with straight edges, each being a little longer than the chord *A C,* lay the one slip upon the other, so that the straight edges being outermost, may intersect each other at *B,* one of the straight edges resting upon a pin at *A,* and the other upon a pin at *C;* fix these two slips together at *B,* and to keep them invariably to the angle, fasten another slip *G H,* at each end, to each of the other two; move the angle *E B F,* so that the side *B E* may slide upon the point *A,* and the side *B F* upon the point C, while a pencil being held to the vertex *B* of the angle, will describe the arc *A B C.*

PLATE 3.

Introduction.

Fig. 1.

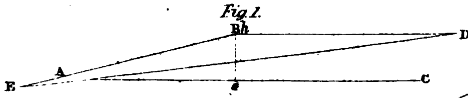


Fig. 2.

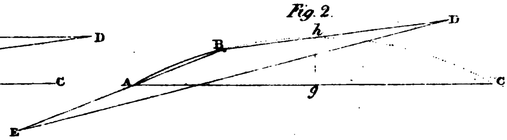


Fig. 3.

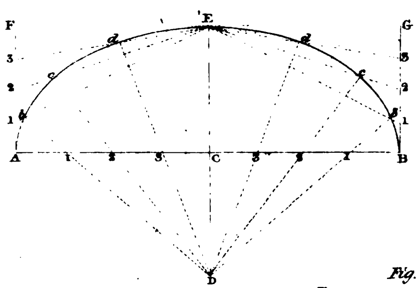


Fig. 4 N.° 1.

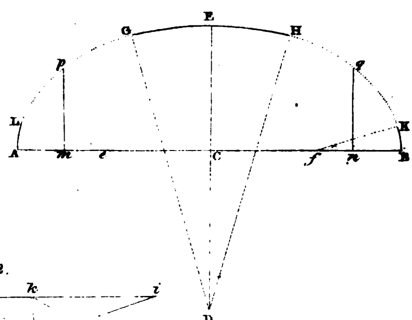


Fig. 4 N.° 2.

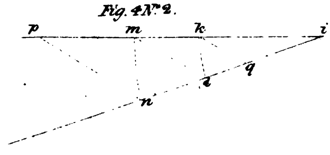


Fig. 5

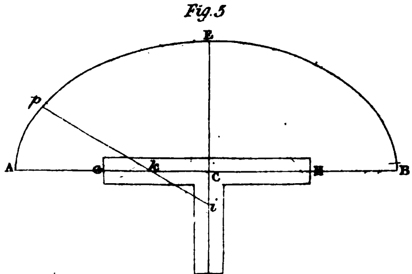
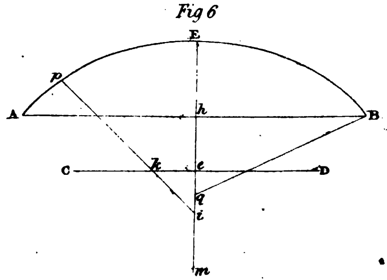


Fig 6



ANOTHER METHOD STILL MORE CONVENIENT.

Let *A C* (Fig. 1) be a straight line equal in length to the chord. Bisect *A C* in *g*, and draw *g h* perpendicular to *A C*, and make *g h* equal to the height of the arc. Draw *h D* parallel to *A C*, and join *h A*, which prolong to any convenient point *E*. Make *h D* equal to *h E*, and join *E D*. Let *A*, *h*, *C*, (Fig. 2), be points at the same distances from one another as the points *A*, *h*, *C* (Fig. 1). In Fig. 2 fix two pins, one in *A*, and the other in *h*, and move the angle *E B D*, so that the edge *B E* may slide upon the pin *A*, and the edge *B D* upon the pin *h*, while during the motion a pencil being held at the point *B* of the angle *E B D*, will describe the arc *A h*. In the same manner, by taking the pin out of *A*, and placing it in *C*, the arc *h C* will be described, which will complete the entire arc *A h C*, of which the chord is *A C*, and the height *g h*.

PROB. VIII.—The two axes of an ellipse being given, to find any number of points in the curve.

Let *A B* (Fig. 3) be the axis-major. Bisect *A B* in *C* by the perpendicular *E D*. Make *C E* equal to the semi-axis minor, and make *C D* equal to *C E*. Through *E* draw *F G* parallel to *A B*, and draw *A F* and *B G* parallel to *D E*. Divide *A F*, *A G*, each into the same number of equal parts, at 1, 2, 3, &c. From the points 1, 2, 3, &c. in *A F*, draw 1 *E*, 2 *E*, 3 *E*, &c. and from *D*, through the points 1, 2, 3, &c. in *A C*, draw *D b*, *D c*, *D d*, &c. meeting the lines 1 *E*, 2 *E*, 3 *E*, &c. in the points *b*, *c*, *d*, &c. In the same manner find the points *b*, *c*, *d*, &c. on the other side of *D E*. From the point *A*, and through the points *b*, *c*, *d*, &c. draw the curve *A b c d E*, and in the same manner draw the curve *B b c d E*.

PROB. IX.—Given the two axes of an ellipse, to find the radius of curvature at the extremities of the axis, thence to describe an approximate curve by the arcs of circles.

Let *A B* (Fig. 4) be the axis-major. Bisect *A B* in *C* by the perpendicular *E D*, and make *C E* equal to the semi-axis minor. In Fig. 4, No. 2, draw *i p* and *i n* at any convenient angle. In *i p* make *i m*, *i k*, respectively equal to *C A*, *C E* (Fig. 4, No. 1). From the point *i* (Fig. 4, No. 2) describe the arcs *m n*, *k l*, meeting *i n* in *n* and *l*, and join *m l*. Draw *n p* parallel to *l m*, and draw *k q* also parallel to *l m*, meeting *i n* in *q*. Make *E D* (No. 1) equal to *i p*, (No. 2), and upon *A B* (No. 1) make *A e*, *B f*, each equal to *i q* (No. 2). From

the centre D (No. 1), with the distance $D E$, describe the arc $G H$, and from the centres e, f, with the distance $A e$ equal to $B f$, describe the arcs $A L$, $B K$. Then the three arcs $A L$, $B K$, $G H$, will very nearly coincide with the curvature of an ellipse drawn upon correct principles. The remaining portions of the curve may be traced by hand. In order to find another point in each curve between the arc thus described, from E, the extremity of the semi-axis minor, with the distance $C A$ or $C B$, the semi-axis major, cut $A C$ at m, and $B C$ at n. Parallel to $C E$ draw $m p$, $n q$; make $m p$, $n q$, each equal to $A e$ or $B f$; and the points p and q are in the curve.

OBSERVATION.

The radius of curvature at the extremity of the semi-axis minor, and at each extremity of the semi-axis major, may be found by calculation, as shown in the observations after Problem XIII.

The points m, n, are the two focii of the semi-ellipse $A E B$, and the lines $m p$, $n q$; the ordinates to the curve passing through the focii are each termed the *latus rectum*, which is a third proportional to the semi-axis major and the semi-axis minor, as shown by the writers on conic sections.

THE USE OF THE TRAMMEL.

The trammel consists of two parts, of which one is fixed and the other moveable. The fixed part is a plate of metal on a thin piece of wood of inconsiderable thickness, comprised between parallel plane surfaces, with two grooves receding from one of the faces at right angles to each other, and the moveable part is a bar having a hole through it at one end to hold a pencil, and two pieces made to slide along its sides, with a steel pin in each slider. These pins are of a cylindrical form, so that the centres of their circular ends, and the point of the pencil, may always be in a straight line, and to admit of being fixed at any distances from the pencil-point, and that the cylindric pins may be fitted exactly, and to move freely in the grooves.

PROB. X.—The axis of an ellipse being given, to describe the curve by a trammel.

Place the middle of the groove $G H$ (Fig. 5) upon the axis-major $A B$, so that the intersection may fall upon the centre C. Let $i p$

be the edge of the trammel bar, and *p* the point of the pencil. Slide each of the moveable pieces, so that their distances from *p* may be respectively equal to *C E*, *C A*. Then place *k*, the centre of the nearest steel-point, in the groove *G H*, and *i*, the centre of the most remote, in the groove *F C*. Move the end *p* of the bar while the points *k*, *i*, remain in the grooves, and the pencil at *p* will describe the curve.

PROB. XI.—Given the semi-axis minor, the abscissa upon the semi-axis minor, an ordinate parallel to the axis-major, to describe the curve.

Let *e E* (Fig 6) be the semi-axis minor, *e* the centre, and *h A* or *h B* an ordinate. Prolong *E e* to *m*, and through *e* draw *C D* parallel to *A B*. From *B*, with the distance *E e*, cut *e D* in *f*; join *B f*, and prolong *B f* to meet *e m* in *q*. In *e C* take *e k* less than *e f*; and from the point *k*, with the distance *f q*, cut *e m* in *i*; join *i k*, and prolong *i k* to *p*; make *k p* equal to *e E* or *f B*; and *p* is the point in the ellipse. In the same manner may be found as many points as will be sufficient to draw the curve, but which may be better drawn by the trammel.

PROB. XII.—To describe a curve comprised by five circular arcs, which shall be a near approximation to the curve of a given semi-ellipse.

Let $A\ G$ (Fig. 1) be the axis-major. Bisect $A\ G$ in M by the perpendicular $D\ J$, and make $M\ D$ equal to the semi-axis minor. Draw $D p$ making any given angle with $D\ J$. From D, with the distance $D M$, cut $D p$ in n, and again from D, with the distance $A M$ or $M G$, cut $D J$ in q, and $D p$ in p. Join $q\ n$, and parallel to $q n$ draw $p\ J$, and draw $M\ r$ parallel to $q\ n$, meeting $D p$ in r. From J, with the radius $J D$, describe the arc $C E$, so that $D\ C$ and $D\ E$ may be each about 15°, or $C E$ about 30°, and join $C J$ and $E J$. In $A\ G$ make $A\ H$ and $G\ L$, each equal to $D\ r$. In $C\ J$ make $C\ s$ equal to $A\ H$, and join $H\ s$. Bisect $H\ s$ by a perpendicular $t\ I$, meeting $C J$ in I. Join $I\ H$, and prolong $I\ H$ to B. In $J\ E$, make $J\ K$ equal to $J\ I$. Join $K L$, and prolong $K L$ to F. From the centre I, with the distance $I\ C$, describe the arc $B\ C$; and from the centre H, with the distance $H\ B$, describe the arc $A\ B$. Also from the centre K, with the distance $K E$, describe the arc $E\ F$; and from the centre L, with the distance $L\ F$, describe the arc $F\ G$; then the curve $A\ B\ C\ D\ E\ F\ G$ will be a very near approximation to the curve of a semi-ellipse.

PROB. XIII.—To describe a curve composed by circular arcs from seven centres, which shall be a very near approximation to the curve of a semi-ellipse, the two axes being given.

Let $A\ I$ (Fig. 2) be the axis-major. Bisect $A\ I$ in J by a perpendicular $E\ N$, and make $J\ E$ equal to the semi-axis minor. By Problem VIII, find the points B, C, D. Bisect $D\ E$ by a perpendicular $t\ N$, and join $D\ N$, and suppose $C\ D$ joined. Bisect $C\ D$ by a perpendicular $u\ M$, meeting $D\ N$ in M, and join $C M$. Suppose $B\ C$ joined, and bisect $B\ C$ by a perpendicular $v\ L$, meeting $C M$ in L. Let $N D$ intersect $A\ J$ in a, and let $M\ C$ intersect $A\ J$ in β. In $J I$ make $J a'$, $J \beta'$, respectively equal to $J\ a$, $J\ \beta$. Join $N\ a'$, and prolong $N\ a'$ to F. In $N\ F$ make $N\ O$ equal to $N\ M$; join $O\ \beta'$, and prolong $O\ \beta'$ to G. In $O\ G$ make $O\ P$ equal to $M\ L$. From the point N, with the radius $N E$, describe the arc $D\ F$; from the point M, with the radius $M D$, describe the arc $C D$. Draw $L\ u$ parallel to $A\ I$. From the centre L, with the distance $L\ C$, describe the arc $u\ C$. Join $u\ A$, and prolong $u\ A$ to meet the arc $u\ C$ in x. Join $x\ L$, intersecting $A\ J$ in K. In $J\ I$ make $J\ Q$ equal to $J K$; join $P\ Q$, and prolong $P\ Q$ to H. From the centre O, with the radius $O\ F$, describe the arc $F\ G$; from the centre P, with the radius $P\ G$, describe the arc $G H$; and from the centre Q, with the radius $Q\ H$, describe the arc $H I$. Then the curve $A\ x\ CDEFGHI$, comprised by the circular arcs $A\ x$, $x\ C$, $C\ D$, &c. will be a very near approximation to the semi-ellipse, of which the axis-major is $A\ I$, and the semi-axis minor $J E$, since the points A, B, C, &c. are in the curve.

Fig. 1.

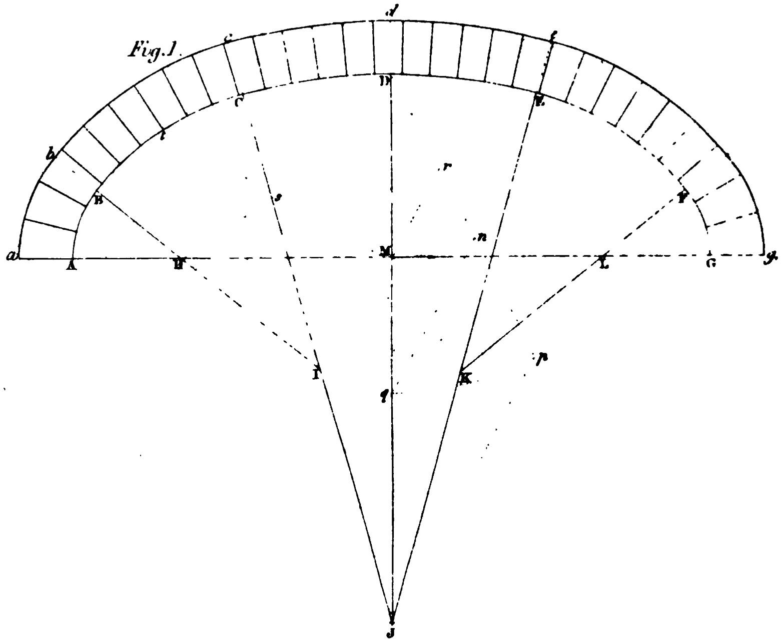
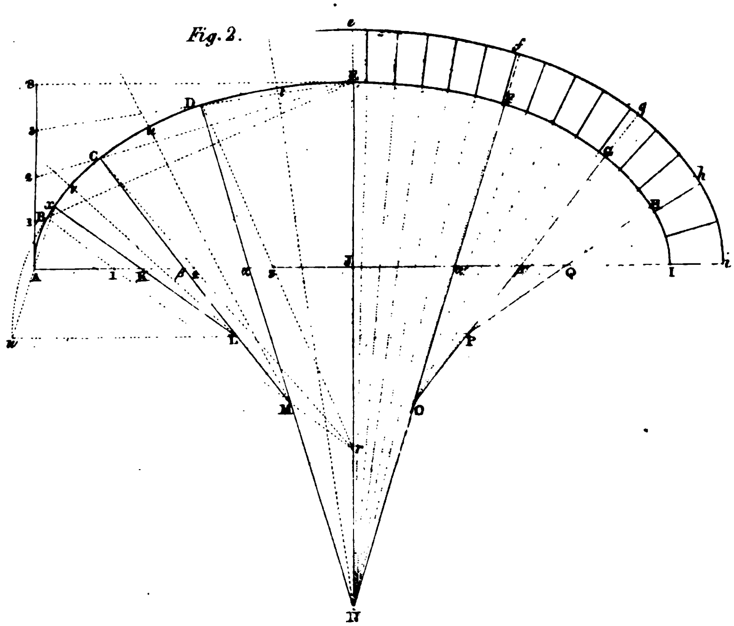


Fig. 2.



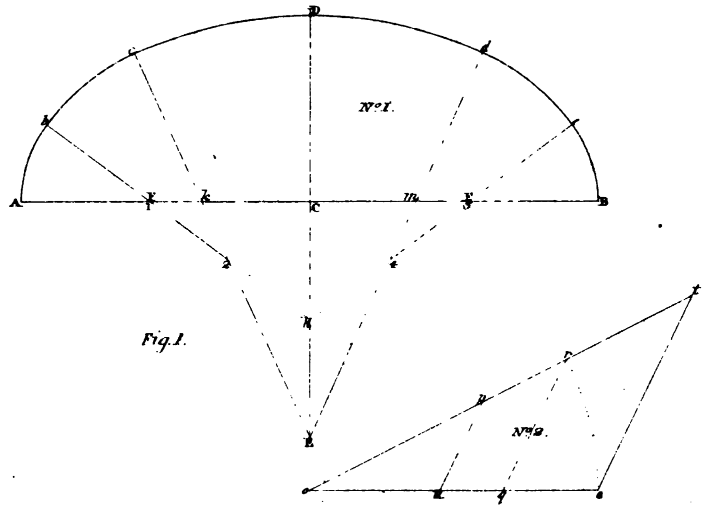
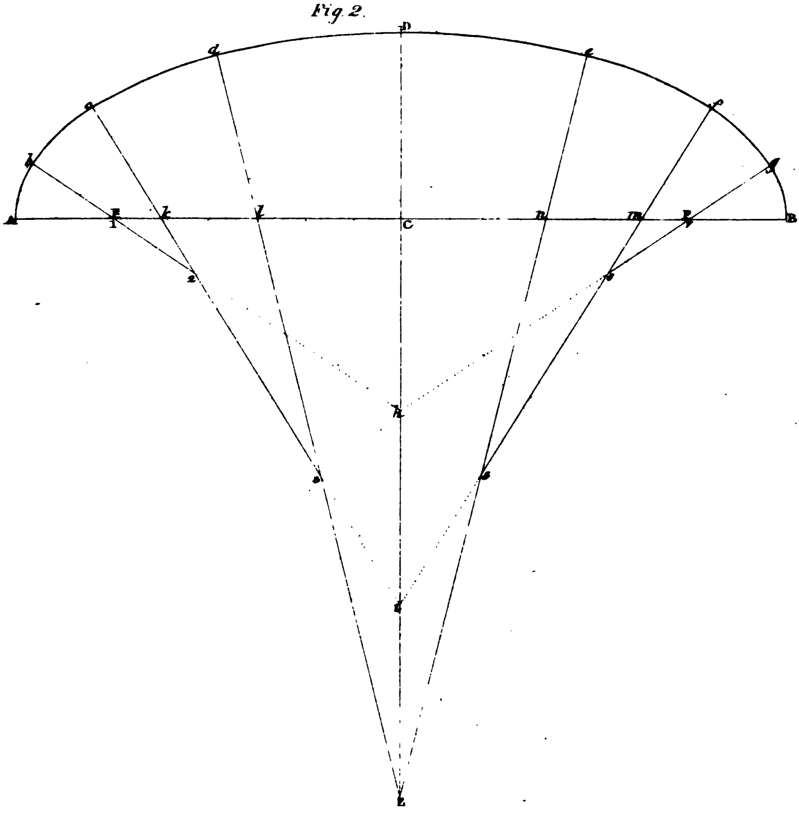


Fig. 1.

Fig. 2.



ANOTHER METHOD FROM FIVE CENTRES.

Bisect *A B* (Fig. 1) in *C*, by the perpendicular *D E*, and make *C D* equal to the semi-axis minor.

Draw two straight lines *o t*, *o s*, (No. 2) forming any convenient angle. In *o t* and *o s*, make *o p*, *o q*, each equal to *C D*, and *o r*, *o s*, each equal to *C A* or *C B*. Join *q r*, and draw *s t* and *p u* parallel to *q r*. Make *D E* equal to *o t*, *A F* and *B F′* each equal to *o u*.

Divide *E C* into two equal parts in *h*, and divide *F C*, *F′ C*, each into three equal parts, making *F k*, *F′ m*, each equal to one part, and *C k*, *C m*, will be each equal to two parts. From *E* through *k* and *m* draw *E c*, *E d*; and from *h*, through *F* and *F′*, draw *h b*, *h e*, intersecting *E c* in 2; and draw *h e*, intersecting *E d* in 4. From *F*, which is the first centre, describe the arc *A b*; from 2, describe the arc *b c*; from *E*, describe the arc *c d*; from 4, describe the arc *d e*; and from *F′*, describe the arc *e B*.

FROM SEVEN CENTRES.

Bisect *A B* in *C* (Fig. 2) by the perpendicular *D E*, and make *C D* equal to the semi-axis minor.

Find the points *F*, *F′*, by finding the radius of curvature at the points *A* and *B*; and find the point *E*, by the radius of curvature at *D*. Thus, let *C D* be 6 feet, and *C A* or *C B* be 12 feet; then, by proportion,

$$C\,B \text{ or } C\,A\ (12) : C\,D\ (6) :: C\,D\ (6) : A\,F = 3.$$
$$C\,D\ (6) : C\,A\ (12) :: C\,A\ (12) : D\,E = 24.$$

Make *D E* equal to 24 feet, and in *A B* make *A F*, *B F′*, each equal to 3 feet. Divide *E C* into three equal parts at *i h*, and divide *F C* as also *F′ C* into three equal parts, each in proportion of 1, 2, 3; so that by dividing *F C* into six equal parts, make *F k* equal to 1, *k l* equal to 2, and *l C* equal to 3; as also make *F′ m* equal to 1, *m n* equal to 2, and *n c* equal to 3. From *E*, through *l* and *n*, draw *E d*, *E e*; from *i* through *k*, draw *i c*, intersecting *E d* in 3; and through *m* draw *i f*, intersecting *E e* in 5; and from *h* through *F*, draw *h b*, intersecting 3 *c* in 2; and through *F′* draw *h g*, intersecting 5 *f* in 6; and the seven centres are *F*, 2, 3, *E*, 5, 6, *F′*. From *F*, with the

radius *F A*, describe the arc *A b*; from 2, with the radius 2 *c*, describe the arc *b c*; from 3, with the radius 3 *c*, describe the arc *c d*; from *E*, with the radius *E d*, describe the arc *d e*, and so on; then the curves described by the circular arcs will be a very near approximation to the semi-ellipse.

In the same manner may a semi-oval be described from nine, eleven, &c. centres, by always dividing *F C*, *F' C*, each into as many parts as there are equal parts in *C E*, in the proportion of the arithmetical progression 1, 2, 3, &c., the first term and the common difference being unity.

OBSERVATIONS.

The method here shown of describing a semi-oval which shall represent a semi-ellipse, is not so near an approximation as that shown page xvi, (Introduction,) Figure 2. This construction is taken from a French publication: "Traité Elementaire de la Coupe des Pierres ou Art du Trait, par Mr. Simonin, Professeur de Mathematiques. Paris, M.DCC.LXXXII.".

PLATE. 6.

Introduction

Fig. 1

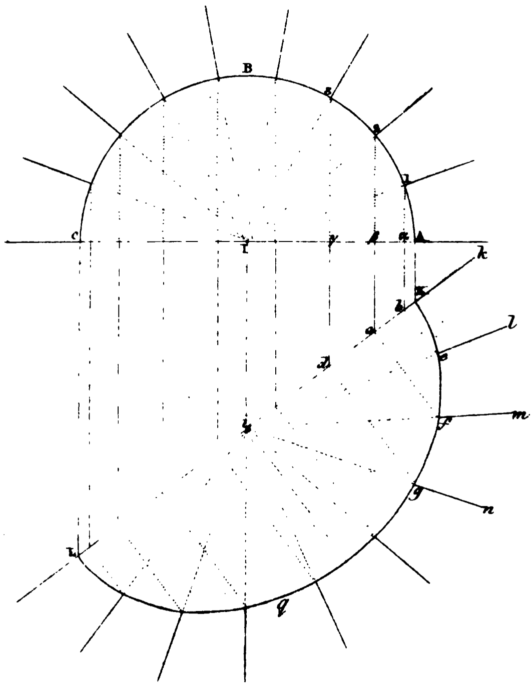
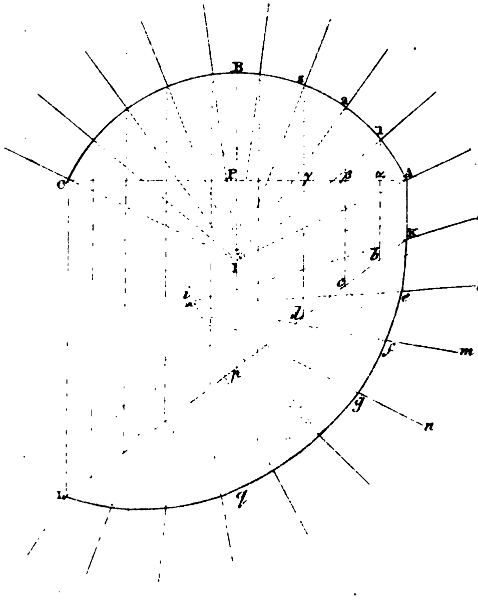


Fig. 2



SECTION II.

ON THE CYLINDER, ITS SECTIONS, AND DEVELOPMENT.

DEFINITIONS.

If a rectangle revolve about one of its sides until the opposite side comes into its first position, the solid comprised is called a *cylinder.*

The fixed line is called the *axis.*

The surface described by the side of the rectangle which is parallel to the axis is called the *cylindric surface,* or sometimes the *curved surface.*

The circles described by the two sides of the rectangle, which are perpendicular to the axis, are called the *bases or ends of the cylinder.*

A portion of a cylinder cut off by a plane parallel to the axis, is called the *segment of a cylinder;* if the plane pass through to the axis, the segment is a *semi-cylinder.*

In this treatise, the segment of a cylinder never exceeds the semi-cylinder, but is generally less than the half.

The plane of the segment of a cylinder, which is parallel to the axis, is called the *springing plane.*

The straight lines in which the springing plane intersects the curved surface of the cylinder, are called the *springing lines.*

The section of the segment of a cylinder, perpendicular to the axis or perpendicular to the springing lines, is called the *right section.*

CONVENTIONAL PRECEPTS AND LEADING PRINCIPLES.

The springing lines are parallel to each other.

A straight line in the chord plane parallel to one of the springing lines, is parallel to the axis of the cylinder; and a straight line which is parallel to the axis of the cylinder, is parallel to the springing lines.

The centering of any arch treated of in this treatise, must be understood to be made in form of the segment of a cylinder, the intrados of the arch being the reverse of the curved surface. The springing plane of the segment is always parallel to the plane of the horizon. The surfaces of the tops or under-edges of the beams to which the ribs are fixed, are placed in the springing plane.

The section of a cylinder cut by a plane perpendicular to the axis, is a circle.

The section of the segment of a cylinder, cut by a plane perpendicular to the axis, is the segment of a circle; and the section of the springing plane is the chord of the arc.

All parallel sections of the segment of a cylinder, cut by a plane either parallel or obliquely to the axis, are equal to one another; and if one was laid upon the other, they would coincide.

If a cylinder be cut by a plane obliquely to the axis, the section is an ellipse of which the longest diameter is the axis-major, and the shortest the axis-minor, and the axis-minor of the ellipse is perpendicular to the axis of the cylinder; and since the two axis of an ellipse are at right angles to each other, if the axis of the cylinder be horizontal, and the axis-minor of the ellipse perpendicular to the horizon, the axis-major of the ellipse shall be parallel to the horizon, and the acute angle made by the axis of the cylinder, and the semi-axis-major of the ellipse, is equal to the angle of inclination which the axis of the cylinder has to the plane of section; therefore, if the cylinder be cut by another plane passing through its axis and through the axis-major of the ellipse, the section of the solid will be a springing plane passing through the axis of a cylinder, and the acute angle made by one of the springing lines, and the axis-major of the ellipse shall be equal to the inclination of the axis of the cylinder to the oblique plane of section.

Hence if a segment of a cylinder be cut by a plane obliquely to the axis, and at the same time perpendicular to the springing plane, the section will still be that of a portion of an ellipse cut off by a double ordinate parallel to the axis-major, and the abscissa a part of the semi-axis minor equal to the height of the right section of the segment of the cylinder.

If a straight line in the chord plane be drawn parallel to the springing lines or the axis of the cylinder, and if two points be taken in this line, two straight lines, drawn from each of the points perpendicular to the chord plane to meet the cylindric surface, are equal to each other.

If the segment of a cylinder be cut by two planes, one perpendicular and the other oblique to the axis, and if a straight line be drawn on the chord plane parallel to the axis to meet the sectional lines, and if from the point of meeting a straight line be drawn in the right section perpendicular to the chord, and in the oblique section perpendicular to the double ordinate, to meet the curved surface, these perpendiculars shall be equal to one another.

In this treatise it must be understood that, when the segment of a cylinder is cut by a plane obliquely to the axis, the plane of section is always perpendicular to the springing plane.

PROB. XIV.—Given the right section of a cylindric arch with radiating joints, the section of the cylindric surface being the arc of a circle not greater than a semi-circle and the angle of obliquity, to find the oblique section.

Let *A B C* (Fig. 1 and Fig. 2) be the right section; draw *C L* and *A K*, each perpendicular to *A C*. In *C L*, take any convenient point *L*, and make the angle *C L K* equal to the angle of obliquity. Divide the arc *A B C* into any number of equal parts at 1, 2, 3, &c., and from the points 1, 2, 3, &c. draw straight lines to radiate to the centre *I*, and these lines will represent the joints. Again, from the points 1, 2, 3, &c., draw 1 *b*, 2 *c*, 3 *d*, &c., intersecting *A C*, perpendicularly in the points α, β, γ, &c., meeting *K L* in *b*, *c*, *d*, &c. Perpendicular to *K L*, draw *b e*, *c f*, *d g*, &c., and make *b e*, *c f*, *d g*, &c., respectively equal to α 1, β 2, γ 3, &c.; and from *K*, through the points *e*, *f*, *g*, draw the curve *K e f g...q...L*, which is the section of the curved surface of the cylinder. In Fig. 1, the arc *A B C* being a semi-circle, the centre *I* is in the diameter *A C*. In Fig. 2, the arc being less than a semi-circle, the centre *I* will fall without the segment; therefore draw *I P* to meet the chord *A C* perpendicularly in *P*, and bisect *K L* in *p*. Draw *p i* perpendicular to *K L*, and make *p i* equal to *P I*. From *i* through the points *K*, *e*, *f*, *g*, &c., draw the radiating lines *K k*, *e l*, *f m*, &c., which are the joint lines of the oblique section.

DEMONSTRATION.

The circular arc *A B C* being the right section of the intrados or concave cylindric surface of the under-side of the arch, and *A C* the chord of the arc *A B C;* draw *A K* and *C L* each perpendicular to *A C*, and let *L K* in the chord plane be the line of section. Then because by construction the straight lines α *b*, β *c*, γ *d*, &c., meeting *A C* in the points α, β, γ, &c., and *K L* in the points *b*, *c*, *d*, &c., are parallel to *C L* or *A K*, and because α 1, β 2, γ 3, &c., are perpendicular to *A C*, and because *b e*, *c f*, *d g*, &c., are perpendicular to *K L*, and because *b e*, *c f*, *d g*, &c., are respectively equal to α 1, β 2, γ 3, &c. Suppose now the arc *A B C* to be raised upon its chord, *A C* in a plane perpendicular to the plane *A C L K*, and suppose the curve *K e f g q...L* to be raised upon the sectional line *K L*, in a plane likewise perpendicular to the chord plane *A C K L*, it is evident that the points *e*, *f*, *g*, &c., are not only in the plane of section, but are also in the surface of the cylinder; for planes passing through α *b*, β *c*, γ *d*, &c., perpendicular to the plane *A C L K*, would cut the segment of the cylinder in rectangles, of which the ends would be the ordinates, viz. *b e* equal to α 1, *c f* equal to β 2, *d g* equal to γ 3, &c.

PROB. XV.—To find the development of the curve of the oblique section of the segment of a cylinder, the angle of obliquity being given.

Let *A B C* (Fig. 1) be a right section of the segment of the cylinder, *A C* being the chord, and *z B* the height of the arc *A B C*. Draw *C H* perpendicular to *A C*, and make the angle *C A H* equal to the complement of the angle of obliquity. Prolong *C A* to *D*, and make *A D* equal to the length of the arc *A B C* (Prob. V, Sec. 1.) Draw *D E* perpendicular to *A D*; make *D E* equal to *C H*, and join *A E*. Divide *A E* into any number of equal parts, at the points 1, 2, 3, &c., and parallel to *D E*, draw 1–1, 2–2, 3–3, &c., meeting *A D* in 1, 2, 3, &c. Divide the arc *A B C* into the same number of equal parts as the straight line *A E*, and from the points 1, 2, 3, &c., in the arc *A B C*, draw the lines 1 a, 2 b, 3 c, &c., intersecting *A C* perpendicularly in *a*, *b*, *c*, &c., and meeting *A H* in a, b, c, &c. From the points 1, 2, 3, &c., in *A D*, make 1 α, 2 β, 3 γ, &c., respectively equal to *a* a, *b* b, *c* c, &c., and from the point *A*, through the points α, β, γ, &c., draw the curve *A m E*, which will be bisected in *m* by the straight line *A E*.

If *H Z* be drawn parallel, and *A Y* perpendicular to *A D*, meeting *H Z* in *Y*, and if *Y Z* be made equal to *Y H*, and if *A Z* be joined, a straight line drawn through the point *m* in the curve will be a tangent.

GENERAL PRINCIPLE.

From this operation it is evident that, in the development of the surface, the semi-ellipse will become an undulated curve of such a nature, that a straight line drawn from one extremity of the curve to the other will divide the curve into two equal parts, the one half being on the one side, and the other half on the other side of the straight line, so that two points being taken in the straight line at equal distances from the point of intersection, shall be equally distant from the curve.

EXAMPLE II—FIGURE II.

Exhibits the development of the curve which is the oblique section of a semi-cylinder, and is described nearly in the same words as in Figure 1, the same letters of reference being used.

OBSERVATION.

In the same manner the development of the curved surface of the segment of a cylinder, cut by a cylindric surface of which the axis is perpendicular to the springing plane, may be found as is evident from Figure 3, and from the explanations given of Figure 1.

It sometimes happens in oblique bridges, that in order to ease the road-way, the ends terminate with circular ends: in such cases this development is useful.

PLATE 7.

Introduction

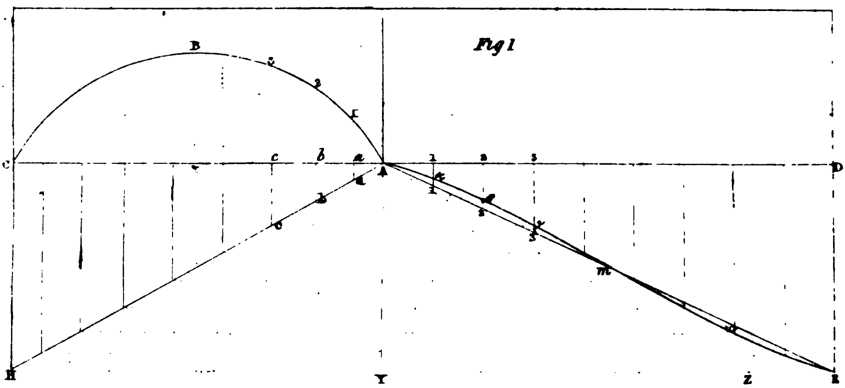


Fig 1

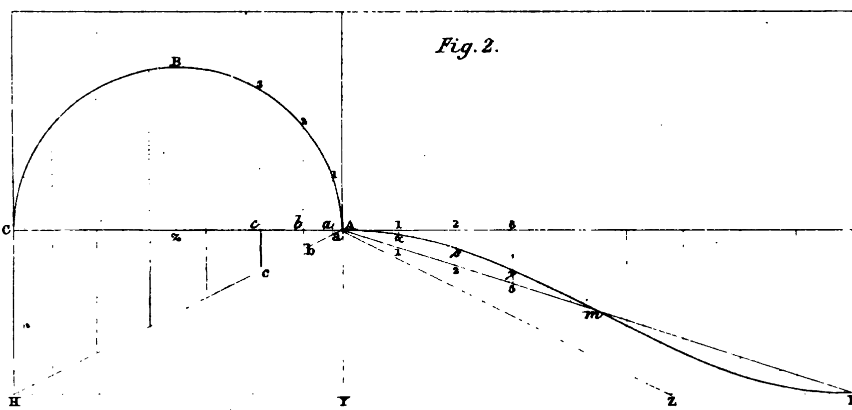


Fig. 2.

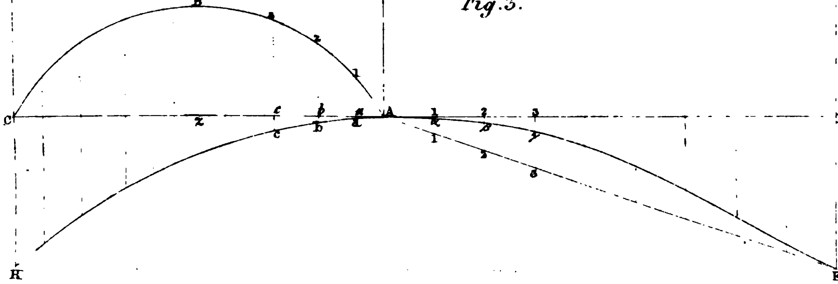
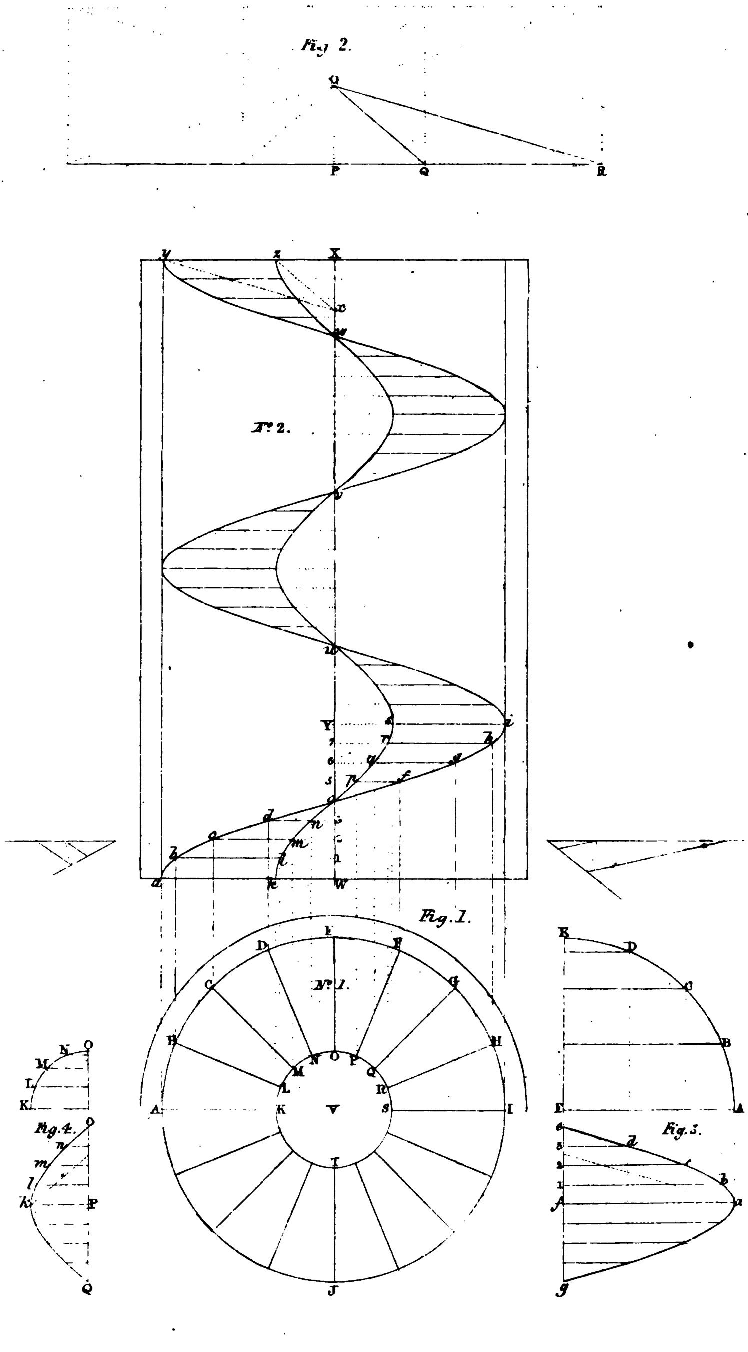


Fig. 3.

PLATE. 8.

Introduction.



SECTION III.—DEFINITIONS.

I. If a limber plane surface, or paper, of which the outline is a right-angled triangle, be rolled upon the curved surface of a cylinder so as to have no vacuity, and that one of the sides about the right angle, not greater than the circumference of the cylinder, may be in a plane perpendicular to the axis, the curve line assumed by the hypothenuse becoming bent, is called a *cylindric spiral.*

From this definition it is obvious, that of the two sides about the right angle of the triangle thus bent, the one which falls upon the circumference will be an arc of a circle, having the same radius as the cylinder, and the other a straight line parallel to the axis.

II. The axis of the cylinder is also called the *axis of the spiral.*

III. The radius of the cylinder is called the *radius of the spiral.*

IV. If the limber surface be unrolled so as to coincide with a plane, the figure again resumes its triangular form, and is called the *triangle of development.*

V. The side of the triangle, which was applied to the circumference of the base of the cylinder, is called the *base of development.*

VI. The side of the triangle which was parallel to the axis of the cylinder, is called the *perpendicular of development.*

VII. The hypothenuse is called the *development of the spiral line.*

VIII. If a straight line be supposed to move perpendicular to the axis of a cylindric spiral, continually touching the spiral, the surface generated is called a *spiral surface,* whether it lies between the axis and the spiral line, or on the other side of the spiral line.

If another cylindric surface, about the same axis, be supposed to intersect the spiral surface, the line of intersection of the two surfaces will be another spiral, of which the triangle of development shall have its perpendicular equal to the perpendicular of the triangle of development of the first spiral, and the lengths of the bases of the two developments will have the same ratio to one another as the radii of their respective cylindric surfaces.

IX. In the triangles of development of the two spirals belonging to the same spiral surface, the difference of the angles made in each by the hypothenuse and perpendicular is called the *angle of the twist.*

ENERAL PRINCIPLES.

If a spiral surface be cut by a plane, either perpendicular to or passing along the axis, the section will be a straight line perpendicular to the axis.

If a cylinder be cut by a plane obliquely to its axis, one straight line on the cutting plane will be perpendicular to, or make a right angle with the axis; and this line is the semi-axis minor of the ellipse, which is the section of the curved surface.

If a spiral surface be cut by a plane parallel to the axis of the cylinder, the section will be a curve, excepting in the case in which the straight line drawn in the cutting plane perpendicular to the axis meets the spiral surface.

If a spiral surface be cut by a plane obliquely to the axis of the cylinder, the section will be a curve of contrary flexure; and if the spiral surface be cut by another plane passing along the axis perpendicular to the first plane, the section which is a straight line will intersect the curve of contrary flexure in the point of retrogression.

PROB. XVI.—To find the projections of one or more revolutions of two spiral lines, which comprise a spiral surface between them, on a plane parallel to the axis of the cylindric surfaces.

Let *A E I J*, *K O S T*, (Fig. 1), be circles, of which their centres is *V*, and their diameters respectively equal to the diameters of the greater and less cylindric surfaces. Divide the circumference of the greater circle into as many equal parts, *A B*, *B C*, *C D*, &c. as the number of points to be found in the projection of each spiral in one revolution. From the points *A*, *B*, *C*, &c. of division, draw lines tending to the centre *V*, meeting the circumference of the less circle in *K*, *L*, *M*, &c. which will also divide its circumference into the same number of equal parts as the greater circumference. In the same straight line, with the centre *V*, draw *W X* to represent the axis. In *W X* make *W Y* equal to the length which the axis will have in half a revolution, and draw *a k* in a straight line with *W*, perpendicular to *W X*. Divide *W Y* at the points 1, 2, 3, &c. into eight equal parts, the number of points to be found in half a revolution;

and through the points 1, 2, 3, &c. draw the lines *b l*, *c m*, *d n*, &c. parallel to *a k*. Draw *A a*, *B b*, *C c*, &c. parallel to *W X*, and from the point *a*, draw a curve through *b*, *c*, *d*, &c. which will be the projection of the exterior cylindric spiral. Draw *K k*, *L l*, *M m*, &c. also parallel to *W X*, and from the point *k*, draw another curve through the points *l*, *m*, *n*, &c. which will be the projection of the spiral of the interior cylindric surface.

In Fig. 2, draw the straight line *P R*, and draw *P O* perpendicular to *P R*. Make *P O* equal to the length of four parts along the axis; make *P R* equal to the arc *A B C D E* of a quarter of the circumference of the exterior cylinder, and draw *O R*. In *P R* make *P Q* equal to the arc *K L M N O* of a quarter of the circumference of the interior cylindric surface, and draw *Q O*; then the angle *Q O R* is the angle of the twist.

The curve *e a g* (Fig. 3), drawn as shown from the adjacent quadrant *A B C D E*, is the same as the curve *u i h g f o* (No. 2) found by projection, and the curve *O k Q*, drawn as shown from the adjacent quadrant *K L M N O*, is the same as the inner curve *u s r g p o*. Each of the curves thus drawn by means of a quadrantal arc, is called the figure of the sines, or a sinical curve; therefore the projection of a spiral line is either a sinical curve, or formed by two or more sinical curves.

OBSERVATIONS.

THE construction of the spiral surface, now explained, will be truly exemplified in a spiral stair, supposing the plan to consist of sixteen steps in the whole circumference. The length of the axis corresponding to this number, will be sixteen times the height of one step. A curve line drawn upon the cylindric surface of the wall, through the points of intersection of the lines in which the sides forming the interior angles of the steps meet each other, will form the spiral line upon the concave cylindric surface of the wall of the stair; and if another concentric cylindric surface, of which the radius is equal to that of the inner circle of the plan, be supposed to exist, a curve line drawn upon the convex surface through the points of intersection of the lines in which every two sides forming the interior angles of the steps meet each other, will form the other spiral line next to the well-hole.

In this manner the spiral pump, the invention of which is attributed to Archimedes, may be constructed.

PROB. XVII.—Given the angle of revolution of a spiral surface, the radius, and the length of each spiral, to find the projection of the spiral surface, and to find the angle of the twist.

Draw the straight line *V A* (Fig. 1). Make *V A* equal to the radius of the outer spiral, and *V K* equal to that of the inner spiral. From the centre *V*, with the radius *V A*, describe the circle *A E I*, and make the angle *A V I* equal to the angle of revolution. From the centre *V*, with the distance *V K*, describe the arc *K O S*, meeting *V I* in *S*. Divide the arc *A E I* into any convenient number of equal parts, as here into eight. In this example the number of parts are even. Through the middle *E* draw *V E*, and draw *P Q* perpendicular to *V E*. Let the points of division in the arc *A E I* be *B*, *C*, *D*, &c. Draw the straight lines *B L*, *C M*, *D N*, &c. radiating to the centre *V*, meeting the inner arc *K O S* in the points *L*, *M*, *N*, &c. Prolong *V E* to *Z*, and make *E Z* equal to the length of the

PLATE 9.

Introduction

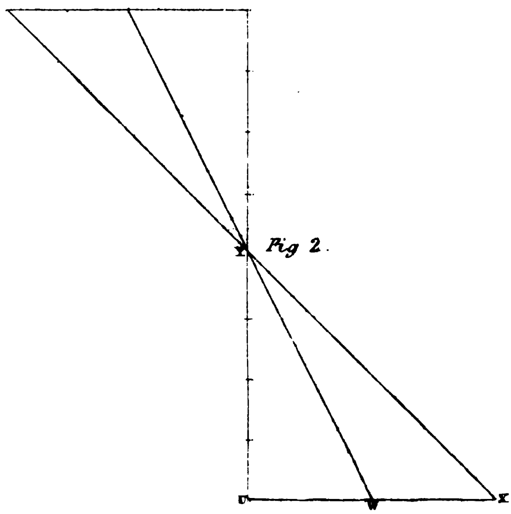


Fig 2.

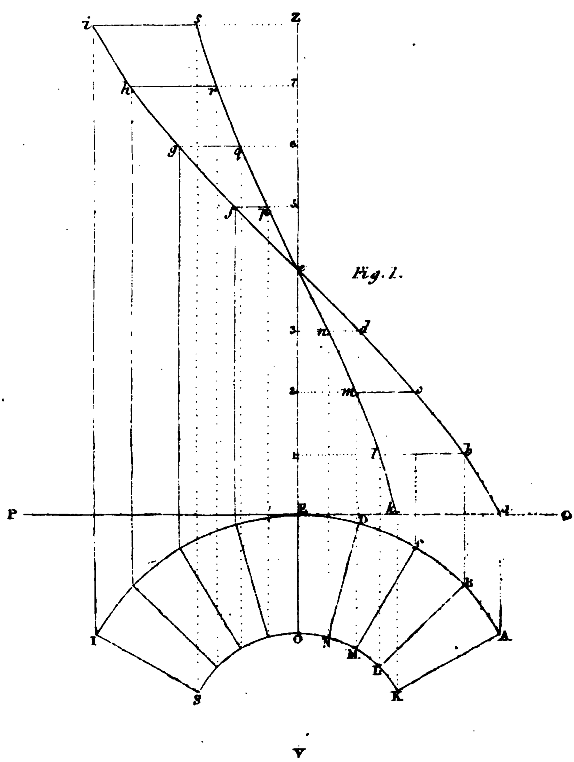


Fig. 1.

spiral. Divide $E\ Z$ into as many equal parts as the arc $A\ E\ I$, viz. eight, and through the points 1, 2, 3, &c. draw the straight lines $l\ b$, $m\ c$, $n\ d$, &c. parallel to $P\ Q$. Draw $A\ a$, $B\ b$, $C\ c$, &c. perpendicular to $P\ Q$, so that $A\ a$ may meet $P\ Q$ in a. Also perpendicular to $P\ Q$ draw $K\ k$, $L\ l$, $M\ m$, &c. so that $K\ k$ may meet $P\ Q$ in k. From the point a, through the points b, c, d, &c. draw the curve line $a\ b\ c\ d\ e\ f\ g\ h\ i$; and from the point k, through the points l, m, n, &c. draw the curve line $k\ l\ m\ e\ p\ q\ r\ s$; and these curves are the projections of the two spirals which comprise the spiral surface.

In Fig. 2, draw $U\ X$ and $U\ Y$ perpendicular to $U\ X$. Make $U\ Y$ equal to four parts of $E\ Z$. Make $U\ X$ equal to the length of the arc $A\ E$, and in $U\ X$ make $U\ W$ equal to the length of the arc $K\ O$. Draw $Y\ X$ and $Y\ W$; then $W\ Y\ X$ is the angle of the twist.

If a straight line were a tangent to the curve $a\ e\ i$, at the point e, (Fig. 1), and another straight line a tangent to the curve $k\ e\ s$ at the same point e, the opposite angles made by these two straight lines would be equal to the angle of the twist.

PLATE 10.

Introduction

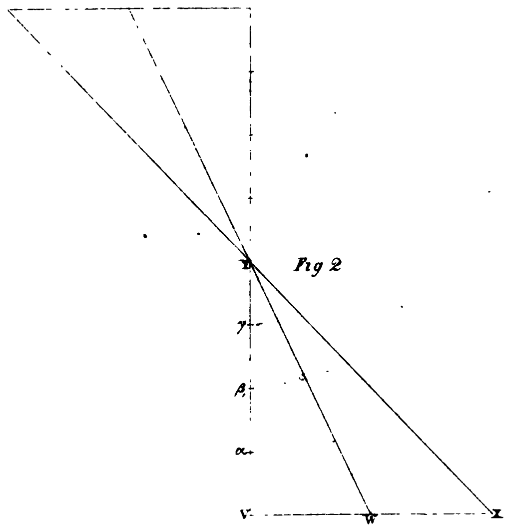


Fig 2

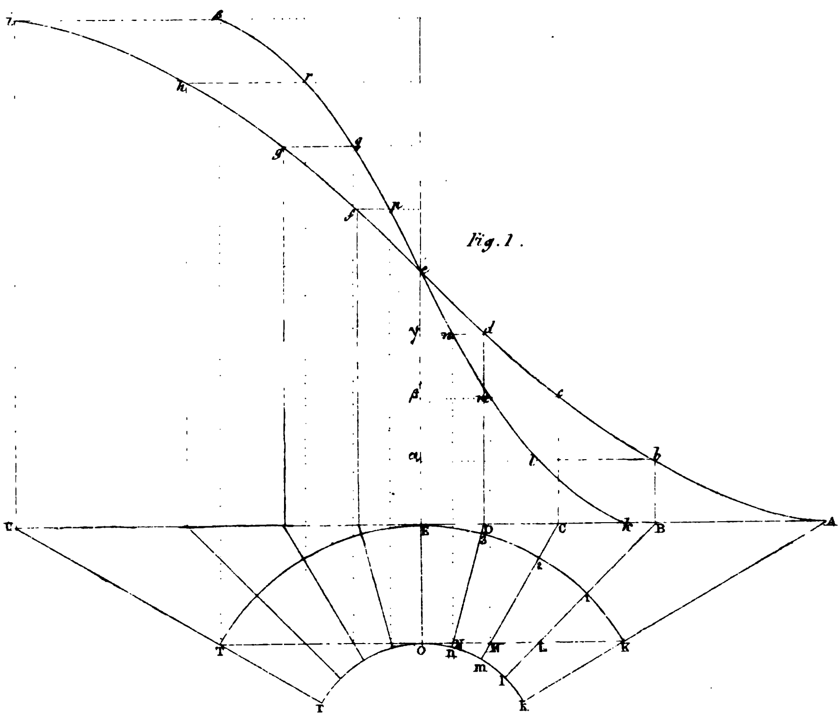


Fig. 1.

S

SECTION IV.—ON THE TRIHEDRAL.

DEFINITIONS.

I. THE solid angle made by three plane angles is called a *trihedral.* Thus, the three faces of a triangular pyramid is a trihedral.

II. The angle made by two plane faces of a solid is called a *dihedral angle.* The measure of a dihedral angle is the angle contained by the straight lines, one drawn upon each face perpendicular to the line in which the two faces meet from the same point.

III. If two of the faces of a trihedral be perpendicular to each other, the trihedral is called a *right trihedral.*

IV. The angle made by the edges of any face is called the *angle of that face.*

V. The two faces which are perpendicular to each other are called the *right faces,* and the remaining face is called the *oblique face.*

VI. The edge between the two right faces is called the *right edge,* and the other two edges are called the *oblique edges.*

If a trihedral be cut by a plane perpendicular to one of its oblique edges, the section shall be a right-angled triangle, and each of the three sectional lines shall be perpendicular to one of the two lines which contain the angle of the face cut by that sectional line, viz. two of the sectional lines shall be perpendicular to the edge to which the cutting plane was perpendicular, and the remaining sectional line upon the opposite face perpendicular to the right edge.

Hence, the three triangles made by the sectional triangle, and the sectional triangle itself, shall be all right-angled triangles.

VII. The oblique edge to which the cutting plane is perpendicular, is called the *adjacent edge.*

VIII. The right face, which has the adjacent edge for one of the lines containing the angle of that face, is called the *adjacent face.*

IX. The face opposite the adjacent edge is called the *opposite face.*

The trihedral consists of five parts, viz. the angles of the three faces and the two acute dihedral angles. Any two of the five parts being given, the three remaining parts may be found, the cases being similar to those of a right-angled triangle.

GENERAL PRINCIPLES.

In every case it is necessary to make one of the right faces the adjacent face. When a dihedral angle is a given or required part, the right face adjacent to the dihedral angle must be the adjacent face, and the other opposite to this angle the opposite face.

The sectional line upon the adjacent face must always be perpendicular to the adjacent edge, and the sectional line upon the opposite face perpendicular to the right edge; moreover, the sectional line upon the oblique face must, as well as the sectional line upon the right face, be perpendicular to the adjacent edge.

If any two parts of a right trihedral be given, the like parts of the sectional triangle may be found; and if the like parts of the sectional triangle be given, the remaining parts of the trihedral may be found.

When the angles of two faces of a trihedral are joined together, so as to make one angle equal to their sum, these angles must either be the angles of the two right faces, or the angles of the oblique and adjacent faces, or the angles of the oblique and opposite faces. In any of the three cases, the radius must always be made upon the connecting line.

It would occupy too much space to explain the properties of all the cases of the trihedral, which are six in number; we shall, therefore, only give the propositions in which the two right faces are concerned, being absolutely necessary.

PROPOSITION I.

The angle of the oblique face is equal to the acute angle of a right-angled triangle, contained by the two lines, of which one is equal to the co-sine of the angle of the adjacent face, and the other equal to the secant of the angle of the opposite face.

PROPOSITION II.

The two sides of the sectional triangle, which contain the right angle, shall be respectively equal to the sine of the angle of the

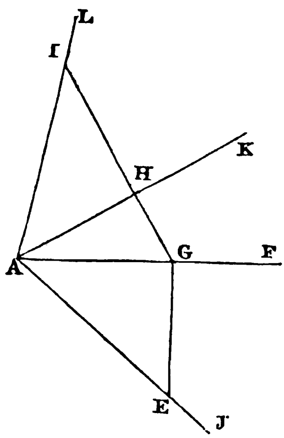
adjacent face, and the tangent of the angle of the opposite face; and the dihedral angle of the trihedral shall be equal to the angle of the sectional triangle, contained by the hypothenuse, and the side equal to the sine of the angle of the adjacent face.

REMARK.

An angle is generally known to the mason by the name of a bevel, and is transferred from one place to another by means of an instrument of the same name, which is so well understood by workmen as not to require description. Bevels are, however, of two kinds, one of which is drawn upon a surface, and the other is the angle made by two surfaces, which we have here called a dihedral angle. It must be observed that, in taking this angle, when the two legs of the bevel are sufficiently extended, the inner angle, in which the inner edges meet, must rest upon the arris of the stone, with one of the inner edges upon the one surface, and the other upon the other surface, while each edge of the bevel is perpendicular to the arris of the stone. The same is to be observed in working one surface, the other being already wrought.

A dihedral angle may be either greater or less than a right angle; but if the acute angle be given, the obtuse angle will be found by deduction or subtraction, as it is the complement of two right angles.

PROB. XIX.—Given the angles of the two right faces of the trihedral, to find the angle of the oblique face.



Let *F A K, F A J,* be the angles of the two right faces, *F A K* being the angle of the adjacent face, and therefore *F A J* the angle of the opposite face.

From any convenient point *G*, in the right edge *A F*, draw *G I* intersecting the adjacent edge *A K* perpendicularly in *H*, and draw *G E* perpendicular to *A F*, meeting *A J* in *E*. From the point *A*, with the distance *A E*, cut *H I* in *I*, and through *I* draw *A L*; then the angle *K A L* or *H A I* is the angle of the oblique face.

For by making *A G* radius, *G H* being perpendicular to *A H*, *A H* is the co-sine of the angle *G A H* of the adjacent face; and *G E* being perpendicular to *A G*, *A E* is the secant of the angle *G A E* of the opposite face. But *A I* is equal to *A E*; therefore the angle *H A I* of the right-angled triangle *A H I* being contained by the sides *A H*, *A I*, respectively equal to the co-sine of the angle of the adjacent face, and the secant of the angle of the opposite face, is equal to the angle of the oblique face.

ILLUSTRATION.

Suppose the plane of the triangle *A G E* to be raised perpendicular to the plane of the triangle *F A K*, and the triangle *H A I* to be turned upon *A H* as a hinge, until the point *I* fall upon *E*, and the straight line *A I* or *A L* upon *A E* or *A J*; and the three plane angles thus united will form the trihedral solid angle, and the ends of the three faces will be a right-angled triangle, and is what is here called the sectional triangle.—*See* the demonstration, Prob. XXI.

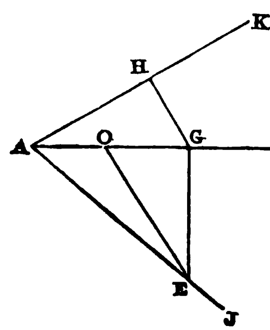
REMARK.

Let *F A K* be the angle which the upper edge of the hip-rafter of a roof makes with its base in a vertical plane, *A K* being the hip-line,

and $A F$ the base or plan; and let $F A J$ be the angle which the base of the hip-line makes with the edge of the wall-plate in a horizontal plane; then $K A L$ is the angle which the hip-line makes with the wall-plate.

The method of finding the angle of any face having the angles of the other two faces given, or of finding a third part, two parts being given, will be found in my Book of Dialling, pages 3, 4, 5, 6, in which the equations are investigated.

PROB. XX.—Given the angles of the two right faces, to find the dihedral angle adjacent to one of them.



Let $F A K$, $F A J$, be the angles of the two right faces, $F A K$ being the angle of the adjacent face, and therefore $F A J$ the angle of the opposite face. From any convenient point G, in the right edge $A F$, draw $G H$ intersecting the adjacent edge $A K$ perpendicularly in H, and draw $G E$ perpendicular to $A F$, meeting $A J$ in E. In $A F$ make $G O$ equal to $G H$; join $O E$; and the angle $G O E$ is the dihedral angle required.

DEMONSTRATION.

For by making $A G$ radius, $G H$ being perpendicular to $A H$, $G H$ is the sine of the angle $F A K$ or $G A H$ of the adjacent face, and $G E$ being perpendicular to $A G$, $G E$ is the tangent of the angle $G A E$ of the opposite face; but $G O$ is equal to $G H$; therefore $G O$ is equal to the sine of the angle of the adjacent face; hence the two sides $G O$, $G E$, which contain the right angle of the sectional triangle $G O E$, are respectively equal to the sine of the angle of the adjacent face, and the tangent of the angle of the opposite face; and the dihedral angle equal to the angle $G O E$ contained by the hypothenuse, and the side $O G$ equal to the sine of the angle of the adjacent face.

OR THUS.

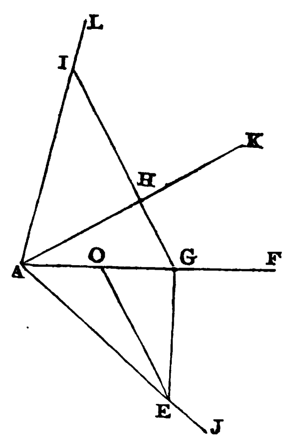
For let the plane containing the angle $G A H$ or $F A K$ be raised upon the straight line $A F$ as a hinge, until it becomes perpendicular

to the plane containing the angle *F A J* or *G A E*, and let the plane containing the triangle *O G E* be made to revolve upon the line *G E* until the side *G O* fall upon *G H;* and because *G O* is equal to *G H*, the point *O* will coincide with the point *H;* therefore the straight line *O E* will coincide with a straight line drawn from *H* to *E;* and because the point *H* is on one edge of the oblique face, and the point *E* on the other edge, the straight line *O E* will be on the oblique face perpendicular to the adjacent edge *A K;* and because *O E* and *H G* are drawn, one in the plane of the oblique face, and the other in the plane of the right face, each perpendicular to *A K*, the line of common section of these two planes, the angle *G O E* is the dihedral angle required.

REMARK.

Let *F A K* be the angle which the upper edge of the hip-rafter of the roof makes with its base in a vertical plane, *A K* being the hip-line and *A F* its base or plan; and let *F A J* be the angle which the base of the hip-line makes with the edge of the wall-plate in a horizontal plane; then *G O E* or *F O E* is the dihedral angle of the back.

PROB. XXI.—Given the angles of the two right faces to find the angle of the oblique face, and the dihedral angle adjacent to either of the right faces, in one operation.



Let *F A K* be the angle of the adjacent face, and therefore *F A J* the angle of the opposite face.

From any convenient point *G*, in the right edge *A F*, draw *G I*, intersecting the adjacent edge *A K* perpendicularly in *H*, and draw *G E* perpendicular to *A F*, meeting *A J* in *E;* from the point *A* with the distance *A E*, cut *H I* in *I;* through *I* draw *A L;* then the angle *K A L* or *H A I* is the angle of the oblique face.

In *A F*, make *G O* equal *G H;* join *O E;* and the angle *G O E* is the dihedral angle required.

DEMONSTRATION.

For $A\,I$ is equal to $A\,E$, and $G\,O$ equal to $G\,H$.

Let $A\,I = A\,E = a$, $G\,E = b$, and $G\,O = G\,H = c$.

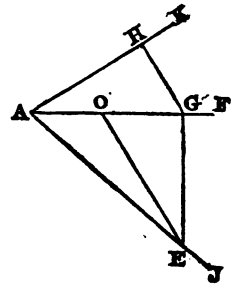
Then by Euclid, Book 1, Proposition 47:

$$A\,G^2 = A\,E^2 - G\,E^2 = a^2 - b^2$$
$$A\,H^2 = A\,G^2 - G\,H^2 = a^2 - b^2 - c^2$$
$$H\,I^2 = A\,I^2 - A\,H^2 = a^2 - (a^2 - b^2 - c^2) = b^2 + c^2$$
$$O\,E^2 = E\,G^2 + O\,G^2 = b^2 + c^2$$

Therefore $H\,I$ is equal to $O\,E$.

Suppose the plane containing the angle $G\,A\,H$ or $F\,A\,K$ to be revolved upon the straight line $A\,F$, until it becomes perpendicular to the plane containing the angle $F\,A\,J$ or $G\,A\,E$, and let the plane containing the angle $O\,G\,E$ be revolved upon the straight line $G\,E$ until the edge $G\,O$ fall upon $G\,H$; and because $G\,O$ is equal to $G\,H$, the point O will coincide with the point H, and the straight line $O\,E$ will, by hypothesis, be perpendicular to $A\,K$ at the point H; moreover, let the plane containing the angle $K\,A\,L$ or $H\,A\,I$ revolve upon $K\,A$, until the edge $A\,I$ or $A\,L$ fall upon the plane containing the angle $F\,A\,J$ or $G\,A\,E$, and because $H\,I$ is perpendicular to $A\,K$, and $O\,E$ is, by hypothesis, also perpendicular to $A\,K$ from the point H, the straight line $H\,I$ will fall upon $O\,E$; and because $H\,I$ is equal to $O\,E$, the point I will fall upon E, and the straight line $A\,I$ upon $A\,E$.

PROB. XXII.—Given a dihedral angle of the trihedral, and the angle of the adjacent face, to find the angle of the opposite face.



Let $F\,A\,K$ be the angle of the adjacent face, $A\,K$ being the adjacent edge, and consequently $A\,F$ the right edge.

In $A\,F$ take any convenient point G, and draw $G\,H$ intersecting $A\,K$ perpendicularly in H. In $A\,F$ make $G\,O$ equal to $G\,H$, and make the angle $G\,O\,E$ equal to the given dihedral angle. Draw $G\,E$ perpendicular to $A\,G$, and through E draw $A\,J$, and the angle $F\,A\,J$ to the angle of the opposite face.

For making *A G* radius, *G H* is the sine of the angle of the adjacent face; but *G O* is equal to *G H*, and the angle *G O E* of the right-angled triangle *O G E* is equal to the dihedral angle of the trihedral; and having the side *G O* and an angle equal to the given dihedral angle of the trihedral, viz. a side and one of the acute angles, a right-angled triangle is easily constructed, as already done in constructing the triangle *O G E;* but *A G* being radius, *G E* is the tangent of the angle *G A E;* therefore the two sides *O G*, *G E*, which contain the right angle *O G E*, are respectively equal to *G H*, the sine of the angle *G A H* of the adjacent face, and to *G E*, the tangent of the angle of the opposite face; therefore the right-angled triangle *O G E* is the sectional triangle.

PROB. XXIII.—Given a dihedral angle of the trihedral, and the angle of the adjacent face, to find the angle of the opposite face and the angle of the oblique face, in one operation.

Let *F A K* be the angle of the adjacent face, *A K* being the adjacent edge, and consequently *A F* the right edge.

In *A F* take any convenient point *G*, and draw *G I* intersecting *A K* perpendicularly in *H;* in *A F* make *G O* equal to *G H*, and make the angle *G O E* equal to the given dihedral angle; draw *G E* perpendicular to *A G*, and through *E* draw *A J;* and the angle *F A J* is the angle of the opposite face.

From *A*, with the distance *A E*, cut *H I* in *I*, and through *I* draw *A L;* and the angle *K A L* is the angle of the oblique face.

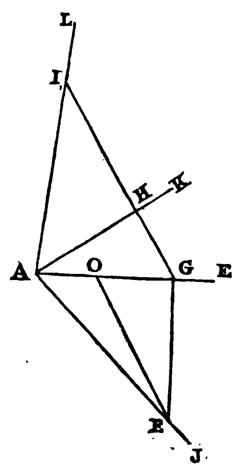
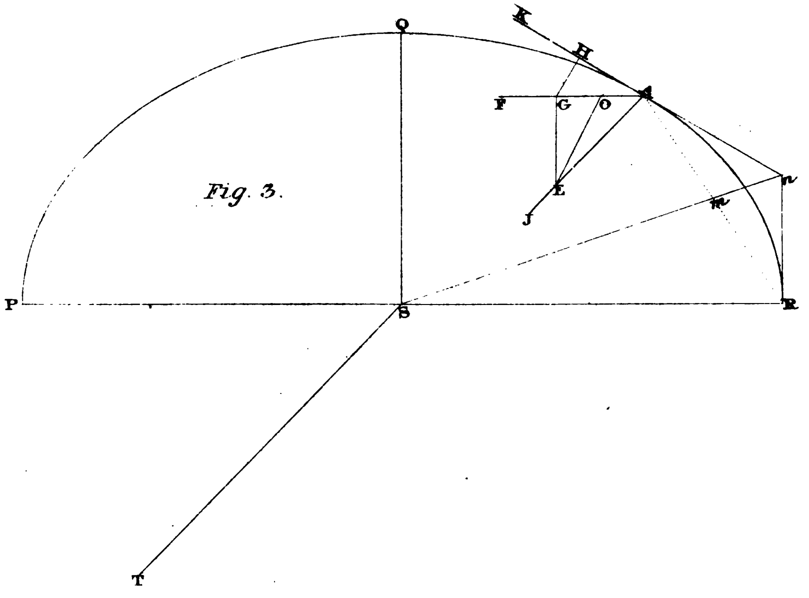


PLATE. II.



SECTION V.

ON THE USE OF THE TRIHEDRAL.

Given a semi-ellipse, which is the section of a semi-cylinder cut by a plane obliquely to the axis, but perpendicularly to the horizontal plane passing through the axis and the angle of obliquity, to find the dihedral angle made by the plane of section, and a plane touching the surface of the cylinder at a given point.

Let *P Q R* be the semi-ellipse, *P R* the axis-major, and *S* the centre of the curve; and let *P S T* be the angle which the axis makes with the plane of section. Make *A K* a tangent to the curve at the point *A*.* Draw *A F* parallel to *P R*, and *A J* parallel to *S T*. Suppose now the plane containing the angle *F A J* to be raised upon the straight line *A F* as a hinge, until it becomes perpendicular to the plane of section *P Q R*; then the straight line *A J* shall be parallel to the axis, and in the curved surface of the cylinder.

It is now evident that the angles *F A K*, *F A J*, may be considered to be the angles of the two right faces of a trihedral, and the angle made by the two right lines *A J*, *A K*, in this position, is the angle of the oblique face, which will touch the surface of the cylinder in the straight line *A J*; moreover, the angle made by the plane *F A K* and the plane of the oblique face shall be the dihedral angle required. Hence we have given the angles *F A K*, *F A J*, of the two right faces of the trihedral, to find the dihedral angle adjacent to the right face *F A K*; we therefore proceed by Prob. XX, thus :—

From any convenient point *G*, in the straight line *A F*, draw *G H* intersecting the adjacent edge *A K* perpendicularly in *H*, and draw *G E* perpendicular to *A F*, meeting *A J* in *E*; in *A F* make *G O* equal to *G H*; join *O E*; and the angle *G O E* is the dihedral angle made by the plane of section, and the plane touching the curved surface of the cylinder at *A*.

* Join *A R*, and bisect *A R* in *m*; draw *R n* perpendicular to *S R*; join *S m*, which prolong to *n*, and join *n A*; and *n A*, or *n A* prolonged, is a tangent to the ellipse at the point *A*.

OBSERVATIONS.

It is evident that towards the middle point Q of the curve, the angle $F A K$ is continually diminishing, and at Q will be nothing; hence the dihedral angle made by a tangent plane at Q, and the plane of section $P Q R$, will be a right angle, and that on the other side of the point Q the dihedral angles continually increase towards P; so that any dihedral angle between P and Q is equal to the supplement of the dihedral angle in the curve $R Q$, at the same distance from R as the point between P and Q is from P.

This problem is applied to the oblique arch for cutting the heads of the stones in the faces of the arch, observing that one of the sides of the angle must be a curve instead of a straight line, which must be a tangent to the curve at the point in which the two lines meet. This will be shown in its proper place.

The trihedral here treated is a pyramid, which has the angle made by two of its faces in planes perpendicular to each other.

If such a trihedral be cut by a plane perpendicular to one of its oblique edges, the section shall be a right-angled *triangle*, and each of the three sectional lines shall be perpendicular to one of the two lines which contain the angle of the face cut by that sectional line, viz. two of the sectional lines shall be perpendicular to the edge to which the cutting-plane was perpendicular, and the remaining sectional line upon the opposite face perpendicular to the right edge.*

* I have called this kind of trihedral a *right trihedral;* but a correspondent, who signs himself W. H. B., in the "Civil Engineer and Architect's Journal," page 152, has erroneously transcribed from a paragraph following Def. 6, page xxiii, Railway Masonry, first edition, "If a trihedral be cut by a plane perpendicular to one of its oblique edges, the section shall be a right angle," leaving out the part that would make sense.

SECTION VI.—PRELIMINARY CALCULATIONS.

TABLE OF THE LENGTHS OF CIRCULAR ARCS.

In the execution of oblique arches, it is necessary to find the development of the intrados; but before this development can be made, it is necessary to find the length of the circular arc, which is a section of the cylindric centre perpendicular to the axis. There are no rules by which the length of a circular arc can be found with sufficient exactness. The following table, which I computed early in the year 1827, contains a series of circular arcs in all proportions— That is to say, if an arc of a circle is required to be executed, which shall have a given chord or span, and a given height not exceeding half the chord, an arc will be found in tables which shall have the same proportion; then, because the corresponding lines of similar figures are proportional, it will be, as the chord of the tabular arc is to the chord of the required arc, so is the length of the curve of the tabular arc to the length of the curve of the required arc. But as the tabular arcs have their chords equal to unity, it will be, as the chord of the given arc is to its height, so is unity, the length of the chord of the tabular arc, to the height of the tabular arc; then as unity, the chord of the tabular arc, is to the length of the tabular arc, so is the given chord to the length of the corresponding arc; and thus we have only to multiply the length of the tabular arc by the given chord, and the product is the length of the arc required. By attending to the following Problem, the greatest exactness will be attained in finding the length of the curve:—

Height of arc.	Length of arc.	Height of arc.	Length of arc.	Height of arc.	Length of arc.	Height of arc.	Length of arc.	Height of arc.	Length of arc.
·080	1.01698	·104	1·02860	·128	1·04313	·152	1·06051	·176	1·08066
·081	1·01741	·105	1·02915	·129	1·04380	·153	1·06130	·177	1·08156
·082	1·01784	·106	1·02970	·130	1·04447	·154	1·06209	·178	1·08246
·083	1·01827	·107	1·03026	·131	1·04515	·155	1·06288	·179	1·08337
·084	1·01871	·108	1·03082	·132	1·04583	·156	1·06368	·180	1·08428
·085	1·01916	·109	1·03139	·133	1·04652	·157	1·06449	·181	1·08519
·086	1·01961	·110	1·03196	·134	1·04721	·158	1·06530	·182	1·08611
·087	1·02006	·111	1·03254	·135	1·04791	·159	1·06611	·183	1·08704
·088	1·02051	·112	1·03312	·136	1·04861	·160	1·06693	·184	1·08797
·089	1·02099	·113	1·03371	·137	1·04932	·161	1·06775	·185	1·08890
·090	1·02146	·114	1·03430	·138	1·05003	·162	1·06858	·186	1·08984
·091	1·02194	·115	1·03490	·139	1·05075	·163	1·06941	·187	1·09079
·092	1·02242	·116	1·03550	·140	1·05147	·164	1·07025	·188	1·09174
·093	1·02291	·117	1·03611	·141	1·05220	·165	1·07109	·189	1·09269
·094	1·02340	·118	1·03672	·142	1·05293	·166	1·07194	·190	1·09365
·095	1·02389	·119	1·03734	·143	1·05367	·167	1·07279	·191	1·09461
·096	1·02440	·120	1·03797	·144	1·05441	·168	1·07365	·192	1·09557
·097	1·02490	·121	1·03860	·145	1·05516	·169	1·07451	·193	1·09654
·098	1·02542	·122	1·03923	·146	1·05591	·170	1·07537	·194	1·09752
·099	1·02594	·123	1·03987	·147	1·05667	·171	1·07624	·195	1·09850
·100	1·02645	·124	1·04051	·148	1·05743	·172	1·07711	·196	1·09949
·101	1·02698	·125	1·04116	·149	1·05819	·173	1·07799	·197	1·10048
·102	1·02752	·126	1·04181	·150	1·05896	·174	1·07888	·198	1·10147
·103	1·02806	·127	1·04247	·151	1·05973	·175	1·07977	·199	1·10247

TABLE OF THE LENGTHS OF CIRCULAR ARCS CONTINUED.

Height of arc.	Length of arc.	Height of arc.	Length of arc.	Height of arc.	Length of arc.	Height of arc.	Length of arc.	Height of arc.	Length of arc.
·200	1·10348	·261	1·17275	·321	1·25539	·381	1·35068	·441	1·45697
·201	1·10447	·262	1·17401	·322	1·25686	·382	1·35237	·442	1·45883
·202	1·10548	·263	1·17527	·323	1·25836	·383	1·35406	·443	1·46069
·203	1·10650	·264	1·17655	·324	1·25987	·384	1·35575	·444	1·46255
·204	1·10752	·265	1·17784	·325	1·26137	·385	1·35744	·445	1·46441
·205	1·10855	·266	1·17912	·326	1·26286	·386	1·35914	·446	1·46628
·206	1·10958	·267	1·18040	·327	1·26437	·387	1·36084	·447	1·46815
·207	1·11062	·268	1·18162	·328	1·26588	·388	1·36254	·448	1·47002
·208	1·11165	·269	1·18294	·329	1·26740	·389	1·36425	·449	1·47189
·209	1·11269	·270	1·18428	·330	1·26892	·390	1·36596	·450	1·47377
·210	1·11374	·271	1·18557	·331	1·27044	·391	1·36767	·451	1·47565
·211	1·11479	·272	1·18688	·332	1·27196	·392	1·36939	·452	1·47753
·212	1·11584	·273	1·18819	·333	1·27349	·393	1·37111	·453	1·47942
·213	1·11690	·274	1·18969	·334	1·27502	·394	1·37283	·454	1·48131
·214	1·11796	·275	1·19082	·335	1·27656	·395	1·37455	·455	1·48320
·215	1·11904	·276	1·19214	·336	1·27810	·396	1·37628	·456	1·48509
·216	1·12011	·277	1·19345	·337	1·27964	·397	1·37801	·457	1·48699
·217	1·12118	·278	1·19477	·338	1·28118	·398	1·37974	·458	1·48889
·218	1·12225	·279	1·19610	·339	1·28273	·399	1·38148	·459	1·49079
·219	1·12334	·280	1·19743	·340	1·28428	·400	1·38322	·460	1·49269
·220	1·12445	·281	1·19887	·341	1·28583	·401	1·38496	·461	1·49460
·221	1·12556	·282	1·20011	·342	1·28739	·402	1·38671	·462	1·49651
·222	1·12663	·283	1·20146	·343	1·28895	·403	1·38846	·463	1·49842
·223	1·12774	·284	1·20282	·344	1·29052	·404	1·39021	·464	1·50033
·224	1·12885	·285	1·20419	·345	1·29209	·405	1·39196	·465	1·50224
·225	1·12997	·286	1·20558	·346	1·29366	·406	1·39372	·466	1·50416
·226	1·13108	·287	1·20696	·347	1·29523	·407	1·39548	·467	1·50608
·227	1·13219	·288	1·20828	·348	1·29681	·408	1·39724	·468	1·50800
·228	1·13331	·289	1·20967	·349	1·29839	·409	1·39900	·469	1·50992
·229	1·13444	·290	1·21102	·350	1·29997	·410	1·40077	·470	1·51185
·230	1·13557	·291	1·21239	·351	1·30156	·411	1·40254	·471	1·51378
·231	1·13671	·292	1·21381	·352	1·30315	·412	1·40432	·472	1·51571
·232	1·13786	·293	1·21520	·353	1·30474	·413	1·40610	·473	1·51764
·233	1·13903	·294	1·21658	·354	1·30634	·414	1·40788	·474	1·51958
·234	1·14020	·295	1·21794	·355	1·30794	·415	1·40966	·475	1·52152
·235	1·14136	·296	1·21926	·356	1·30954	·416	1·41145	·476	1·52346
·236	1·14247	·297	1·22061	·357	1·31115	·417	1·41324	·477	1·52541
·237	1·14363	·298	1·22203	·358	1·31276	·418	1·41503	·478	1·52736
·238	1·14480	·299	1·22347	·359	1·31437	·419	1·41682	·479	1·52931
·239	1·14597	·300	1·22495	·360	1·31599	·420	1·41861	·480	1·53126
·240	1·14714	·301	1·22635	·361	1·31761	·421	1·42041	·481	1·53322
·241	1·14831	·302	1·22776	·362	1·31923	·422	1·42222	·482	1·53518
·242	1·14949	·303	1·22918	·363	1·32086	·423	1·42402	·483	1·53714
·243	1·15067	·304	1·23061	·364	1·32249	·424	1·42583	·484	1·53910
·244	1·15186	·305	1·23205	·365	1·32413	·425	1·42764	·485	1·54106
·245	1·15308	·306	1·23349	·366	1·32577	·426	1·42945	·486	1·54302
·246	1·15429	·307	1·23494	·367	1·32741	·427	1·43127	·487	1·54499
·247	1·15549	·308	1·23636	·368	1·32905	·428	1·43309	·488	1·54696
·248	1·15670	·309	1·23780	·369	1·33069	·429	1·43491	·489	1·54893
·249	1·15791	·310	1·23925	·370	1·33234	·430	1·43673	·490	1·55090
·250	1·15912	·311	1·24070	·371	1·33399	·431	1·43856	·491	1·55288
·251	1·16033	·312	1·24216	·372	1·33564	·432	1·44039	·492	1·55486
·252	1·16157	·313	1·24360	·373	1·33730	·433	1·44222	·493	1·55685
·253	1·16279	·314	1·24506	·374	1·33896	·434	1·44405	·494	1·55884
·254	1·16402	·315	1·24654	·375	1·34063	·435	1·44589	·495	1·56083
·255	1·16526	·316	1·24801	·376	1·34229	·436	1·44773	·496	1·56282
·256	1·16649	·317	1·24946	·377	1·34396	·437	1·44957	·497	1·56481
·257	1·16774	·318	1·25095	·378	1·34563	·438	1·45142	·498	1·56680
·258	1·16899	·319	1·25243	·379	1·34731	·439	1·45327	·499	1·56879
·259	1·17024	·320	1·25391	·380	1·34899	·440	1·45512	·500	1·57079
·260	1·17150								

PROB. XXIV.—To find the length of the arc of a circle, the chord and height of the arc being given, but the height not to exceed half the chord.

Divide the height of the arc by the chord to three places of decimals; look in the table under "height of arc" for the number which is equal to the quotient; from the next column under "length of arc" take out the opposite number, which will be the length of a similar arc to that which is to be found, having unity for its chord; multiply the tabular length of the arc by the given chord; and the product is the length of the arc to that chord nearly.

But if, after having divided the given height of the required arc by its chord, the quotient does not terminate in the third place of decimals, continue, if necessary, to find three more places of decimals; look in the table under "height of arc" for a number equal to the three first quotient figures; in the column on the right take out the opposite numbers corresponding to the first three figures and to the next greater; multiply the difference by the three remaining figures considered as a decimal; to the first three add the product to the length of the arc corresponding to the three first figures; multiply the sum by the given chord; and the product is the length of the arc required.

EXAMPLE I.

Required the length of an arc, of which the chord is 22 feet and the height 5 feet 6 inches?

22f.$\times$12=264 inches, and (5ft. 6in.)$\times$12=66 inches, 66$\div$264=·250 exactly.

The tabular length of the arc answering to the height ·250 is 1·15912, and 22$\times$1·15912=25·50064, the length of the arc.

EXAMPLE II.

Required the length of an arc, of which the chord is 38ft. 9in. and the height 6 feet?

Here 38ft. 9in.=465 inches, and 6ft.=72 inches, and 72$\div$465= ·154|838.

The tabular length of the arc answering to 155, is 1·06288
and the tabular length of the arc answering to 154, is 1·06209
Their difference is ·00079

·00079$\times$·838=·00066, retaining only five places of decimals,
and 1·06209$\times$·00066=1·06275,
and 465$\times$1·06275$\div$12=41·18156 feet, the answer.

EXAMPLE III.

Required the length of an arc, of which the chord is 22f. 10′ and the height 8 feet?

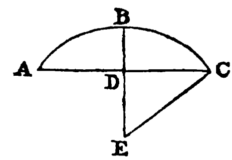
22f. 10′=274 inches, and 8f.=96 inches, and 96÷274=·350|364.
The tabular length of the arc answering to the height 351, is 1·30156
and the tabular length of the arc answering to the height 350, is 1·29997
Their difference is ·00159

·00159×·364=·00058, retaining five places of decimals,
and 1·29997+·00058=1·30055,
and lastly, 274×1·30055÷12=29·69589 feet.

EXAMPLES FOR PRACTICE.

1. Required the length of an arc, of which the chord is 23 feet and the height 6 feet? *Ans.* 26·96934 *feet.*
2. Required the length of an arc, of which the chord is 48 feet 9 inches, and the height 14 feet 6 inches? *Ans.* 59·53496 *feet.*
3. Required the length of an arc, of which the chord is 17·374 feet and the height 6 feet? *Ans.* 22·45815 *feet.*
4. Required the length of an arc, of which the chord is 15·2708 and the height $4\frac{1}{6}$ feet? *Ans.* 18·14155 *feet.*
5. Required the length of an arc, of which the chord is 19 feet and the height 6 feet? *Ans.* 23·68426 *feet.*
6. Required the length of an arc, of which the chord is 45·96 feet, and the height 14 feet 6 inches? *Ans.* 57·3204 *feet.*
7. Required the length of an arc, of which the chord is 19 feet, and the height 7 feet 1 inch? *Ans.* 25·40262 *feet.*
8. Required the length of an arc, of which the chord is 19 feet and the height of the arc 6·8776? *Ans.* 25·06461 *feet.*

PROB. XXV.—Given the chord and height of an arc to find the radius of the circle.



Let *A B C* be the arc, *A C* the chord, and *E* the centre. Draw the radius *E B*, bisecting the chord *A C* perpendicularly in *D*, and join *C E*.

Let $h = DC$ or DA=half the chord, and $a = DB$, the height of the arc; and let $x = EC = EB$, the radius of the circle; then will $DE = x - a$.

Then by Euclid, Book I, Proposition 47 :—

$$EC^2 = ED^2 + CD^2$$

That is, $x^2 = (x - a)^2 + h^2$

Or $x^2 = x^2 - 2a\,x + a^2 + h^2$

Whence $x = \dfrac{a^2 + h^2}{2a} = \frac{1}{2}\left(\dfrac{h^2}{a} + a\right)$

RULE IN WORDS.

Divide the square of the half-chord by the height of the arc; add the height of the arc to the quotient; and half the sum is the radius of the circle.

EXAMPLE.

Given the chord 22 feet, and the height of the arc 5 feet 6 inches, to find the radius of the circle.

Here $22 \div 2 = 11$ feet, the half-chord, and 5ft. 6in.=5·5 feet.

$11^2 = 121$, which, divided by 5·5 feet, gives 22 for the quotient, 22+5·5, and 27·5, and 27 5÷2=13·75=13 feet 9 inches, the radius.

EXAMPLES FOR PRACTICE.

1. Required the radius of an arc, of which the chord is 38 feet 9 inches, and the height 6 feet? *Ans.* 34·2825 *feet.*

2. Required the radius of an arc, of which the chord is 22 feet 10 inches, and the height 8 feet? *Ans.* 12·1462 *feet.*

3. Required the radius of an arc, of which the chord is 23 feet and the height 6 feet? *Ans.* 14·02083 *feet.*

4. Required the radius of an arc, of which the chord is 48 feet 9 inches, and the height 14 feet 6 inches? *Ans.* 27·7376 *feet.*

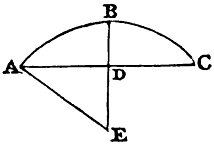
5. Required the radius of an arc, of which the chord is 17·374 feet and the height 6 feet? *Ans.* 9·2886 *feet.*

6. Required the radius of an arc, of which the chord is 15·2708 feet and the height $4\frac{1}{6}$ feet? *Ans.* 9·0792 *feet.*

7. Required the radius of an arc, of which the chord is 19 feet and the height 6 feet? *Ans.* 10·52008 *feet.*

8. Required the radius of an arc, of which the chord is 45·96 feet, and the height 14 feet 6 inches? *Ans.* 25·4596 *feet.*

Prob. XXVI.—Given the radius of a circle, to find the height of an arc of that circle, the chord of the arc being given.



Draw the straight line $A\ C$ equal to the given chord; from the points A and C, with the length of the given radius, describe arcs intersecting each other in E; and draw the radius $E\ B$ bisecting $A\ C$ perpendicularly in D; then $D\ B$ is the height of the arc.

BY CALCULATION.

Join $E\ A$, and let $E\ A = E\ B = r$, $A\ D = C\ D = h$, and $D\ B = x$; then will $D\ E = r - x$.

By Euclid, Book I, Proposition XLVII:—

$$A\ D^2 + D\ E^2 = E\ A^2,$$

Or $h^2 + (r-x)^2 = r^2$,

Or $(r-x)^2 = r^2 - h^2$,

Hence, $r - x = \sqrt{(r^2 - h^2)}$,

By changing the signs, $x - r = -\sqrt{(r^2 - h^2)}$,

therefore, $x = r - \sqrt{(r^2 - h^2)}$.

ARITHMETICAL RULE.

From the radius subtract the square root of the difference of the square of the radius, and that of the half-chord, and the remainder is the height of the arc.

EXAMPLE I.

Given the radius of a circle equal to 36 feet, and the chord of an arc of that circle equal to 4 feet, to find the height of the arc.

Here $36^2 = 1296$, the square of the radius,

And $2^2 = \quad 4$, the square of the half-chord,

1292, the difference of those squares.

$\sqrt{(1292)} = 35{\cdot}9444$, the square root of the difference,

$36 - 35{\cdot}9444 = 0{\cdot}0556$, the height of the arc.

EXAMPLE II.

Given the radius of a circle equal to 726 feet, and the chord of an arc of that circle equal to 4 feet, to find the height of the arc.

Here $726^2 = 527076$, the square of the radius,

And $2^2 = \quad 4$, the square of the half-chord,

527072, the difference of their squares,

$\sqrt{(527072)} = 725{\cdot}9972$, the square root of the difference.

$720 - 725{\cdot}9972 = 0{\cdot}0028$, the height of the arc.

EXAMPLE FOR PRACTICE.

Required the height of an arc of a circle of which the radius is 10 feet, to a chord of 19 feet? *Ans.* 6·87751.

Let the parallelogram $A\ G\ Q\ H$ be the plan of an oblique arch. Two of the opposite sides, $A\ H$, $G\ Q$, are the lengths of the faces upon which the arch-way terminates, and the other two, $A\ G$, $H\ Q$, are the lengths of the springers or abutments. The acute angle $A\ H\ Q$, or $A\ G\ Q$, is that which is called the angle of obliquity. If a straight line be drawn from the vertex A of one of the obtuse angles, to meet the opposite springing line $H\ Q$, or $H\ Q$ prolonged perpendicularly in C, the line $A\ C$ is the width of the arch-way, and the line $C\ H$ is called the distance of obliquity; for the less $C\ H$, the angle $A\ H\ Q$ will be nearer to a right angle, and when $C\ H$ is nothing, the angle $A\ H\ Q$ will be a right angle, and consequently $A\ H$ will coincide with $A\ C$. A straight line $H\ R$ drawn to meet the opposite side $G\ Q$, prolonged perpendicularly in R, is the distance between the two faces, or the distance between the front and rear elevations of the arch.

PROB. XXVII.—Given the width of the arch-way, the angle of obliquity, and the distance between the two faces, to draw the plan of the arch.

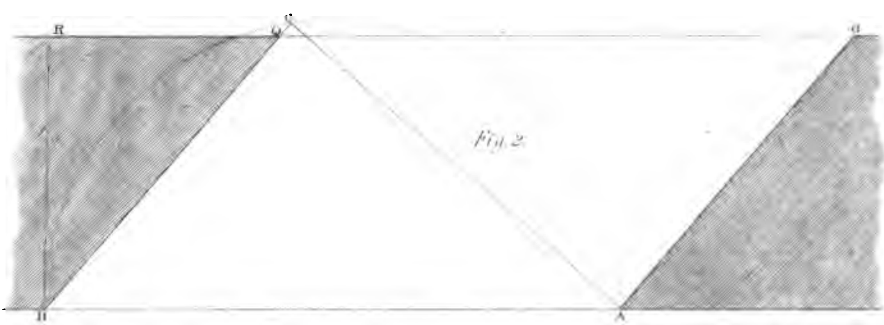
Draw the straight line $A\ C$, and make $A\ C$ equal to the width of the arch-way. Make the angle $C\ A\ H$ equal to the complement of the angle of obliquity; thus, if the angle $A\ H\ C$ be 50°, make $C\ A\ H$ equal to 40°. Draw $C\ H$ perpendicular to $A\ C$, and $H\ R$ perpendicular to $A\ H$. Make $H\ R$ equal to the distance between the two faces of the arch, and draw $R\ G$ parallel to $A\ H$, intersecting $H\ C$ prolonged to Q. Draw $A\ G$ parallel to $H\ Q$; and the parallelogram $A\ G$, $Q\ H$, is the plan of the aperture, $A\ G$, $H\ Q$, being the two springing lines, $A\ H$, $G\ Q$, the lengths of the faces.

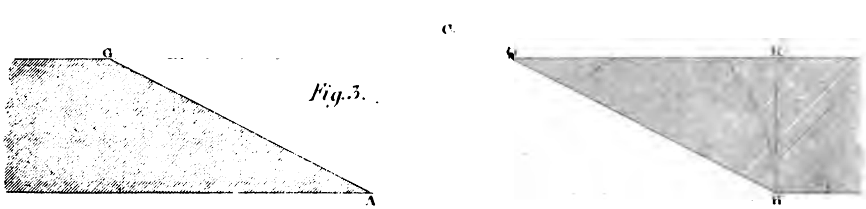
PROB. XXVIII.—Given the length of one of the faces, the angle of obliquity, and the distance between the two faces, to draw the plan of the arch-way.

Draw the straight line $A\ H$, and make $A\ H$ equal to the length of the given face. Make the angle $A\ H\ Q$ or $A\ H\ C$ equal to the angle of obliquity. Upon $A\ H$ as a diameter, describe the semi-circle $A\ C\ H$ intersecting $H\ Q$, or $H\ Q$ prolonged in C, and join $A\ C$, $C\ H$.

PLATE.12

Introduction





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Draw *H R* perpendicular to *A H*, and make *H R* equal to the distance between the two faces. Draw *G R* parallel to *A H*, and *A G* parallel to *H Q*, and *A G*, *Q H*, is the plan of the aperture of the arch.

PROB. XXIX.—Given the width of the arch-way, the distance of obliquity, and the distance between the two faces, to draw the plan of the arch.

Draw the straight line *A C*, and make *A C* equal to the width of the arch-way. Draw *C H* perpendicular to *A C*, make *C H* equal to the distance of obliquity, and join *A H*. Draw *H R* perpendicular to *A H*, and make *H R* equal to the distance between the two faces. Through *R* draw *G R* parallel to *A H* intersecting *H C*, or *H C* prolonged in *Q*. Draw *A G* parallel to *H Q*; and *A G*, *Q H*, is the plan.

PROB. XXX.—Given the width of the arch-way, the length of one of the faces, and the distance between the two faces, to draw the plan of the arch.

Draw the straight line *A H*, and make *A H* equal to the length of the given face. Upon *A H* as a diameter, describe the semi-circle *A C H*. From the point *A*, with the width of the arch-way, cut the arc in *C*, and join *A C*, *C H*. Draw *H R* perpendicular to *A H*, and *H R* equal to the distance between the two faces. Draw *G R* parallel to *A H*, intersecting *H C*, or *H C* prolonged in *Q*. Draw *A G* parallel to *H Q*; and *A G*, *Q H* is the plan.

EXAMPLES FOR PRACTICE.

1. There is an oblique arch, of which the angle of obliquity is 76° 42′, the width of the arch-way 38·75 feet, the distance between the faces 26 feet, the length of one of the faces 39·82 feet; to draw the plan of this arch, when its width, its angle of obliquity, and the distance between its faces are given, and when its angle of obliquity, the length of one of its faces, and the distance between them are given, and when the width of its arch-way, the distance of its obliquity, and the distance between its faces are given.

2. There is an oblique arch of which the angle of obliquity is 50°, the length of one of the faces 60 feet, the width of the arch-way 45·96 feet, and the distance between the two faces 28·75 feet; to draw the plan of this arch when its angle of obliquity, the length of one of its faces, and the distance between them are given, and when the width of its arch-way, the length of one of its faces, and the distance between them are given.

3. There is an oblique arch of which the angle of obliquity is 26° 54′, the length of one of the faces 42 feet, the width of the arch-way 19 feet, and the distance between the two faces 14 feet; to draw the plan of this arch when its angle of obliquity, the length of one of its faces, and the distance between them are given, and when the width of its arch-way, the length of one of its faces, and the distance between them are given.

It will give great facility to the student to exercise himself in drawing the plans of oblique arches to the dimensions given in the first, second, and third of these examples, to a large scale, say one quarter inch to the foot, or $2\frac{1}{2}$ inches to 10 feet.

Figures 1, 2, 3, in order to be contained in the plate, are drawn to a scale of which every half-inch contains 10 feet; figure 1 is drawn according to the first of these proportions, figure 2 to the second, and figure 3 to the third.

Figure 1, plan of the oblique arch at Gateshead, upon the Brandling Junction Railway.

Figure 2, plan of one of the oblique arches of the bridge over the river Tees, at Croft, on the Great North of England Railway.

Figure 3, plan of the oblique arch over the river Gaunless, near Hagger Leazes Lane.

SECTION V.

ON THE MENSURATION OF THE SIDES AND ANGLES OF A RIGHT-ANGLED TRIANGLE, TWO PARTS BEING GIVEN.

THIS may be divided into three cases. First, when the two sides are given, to find the third; when the two sides are given, to find the angles; and, thirdly, when a side and an angle are given, to find the remaining sides.

PROB. XXXI.—Given any two sides of a right-angled triangle ABC, to find the remaining side.

By Euclid, Prop. XLVII, Book 1, we have

$$AC^2 = AB^2 + BC^2$$

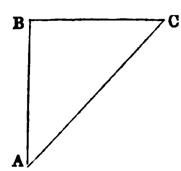
Or $AB^2 + BC^2 = AC^2$;

Therefore $BC^2 = AC^2 - AB^2$,

And $AB^2 = AC^2 - BC^2$;

Hence $BC = \sqrt{(AC^2 - AB^2)}$

And $AB = \sqrt{(AC^2 - BC^2)}$



EXAMPLE.

Let AC be equal to 42 feet, and BC be equal to 19 feet, to find AB.

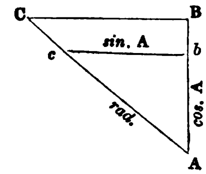
Here, $AB = \sqrt{(42^2 - 19^2)} = \sqrt{(1403)} = 37{\cdot}4566$.

Each of the three straight lines which comprise the surface of a triangle is called a side of that triangle. In a right-angled triangle, the two sides which contain the right angle are called the legs, of which the one is opposite to one of the acute angles, and the other adjacent to the same angle.

On the method of finding a side of a right-angled triangle, a side and an angle being given, or to find the angles when two sides are given.

The trigonometrical table contains a series of numbers, amongst which will be found three numbers in the same ratio as the sides of

any proposed right-angled triangle; and if two acute angles, one in the tabular triangle, and the other in the proposed triangle, be equal, the sides about the equal angles will be proportional; and, reciprocally, if two sides of the proposed triangle be proportional to two sides of the tabular triangle, the angles contained by these two sides of the proposed triangle shall be equal to the angle contained by the corresponding sides of the tabular triangle. The three sides of the tabular triangle are called the radius, sine, and co-sine; the hypothenuse is called the radius, which, in the table of natural sines, is equal to 1 or unity; therefore each of the other two sides must be less than 1. The leg of the tabular triangle which is opposite to an angle, is called the sine of that angle, and the leg of the tabular triangle which is adjacent to an angle, is called the co-sine of that angle. The tabular triangle which is similar to a proposed triangle, will be recognised when an angle of the proposed triangle is given. For instance, let the angle of a triangle be 36°, then the two sides of the tabular triangle which contain that angle are the radius equal to 1, and the co-sine equal to ·80902; therefore, when one side of the proposed triangle is given, the other can be found; and, moreover, the tabular triangle, which is similar to a proposed triangle, will be recognised when two sides of the proposed triangle are given. Suppose the hypothenuse and a leg opposite an angle to be given. In the triangle *A B C*, let the side *B C* be the leg opposite the angle *A*, and *A B* the side adjacent to the angle *A*. Now, then, writing *rad.*=1 for radius, *sin.* for sine, and *cos.* for co-sine,



$$A\,C : B\,C :: \text{rad.} : \text{sin.}\ A = \frac{B\,C}{A\,C}, \text{ because rad.}=1.$$

$$A\,C : A\,B :: \text{rad.} : \text{cos.}\ A = \frac{A\,B}{A\,C}$$

Therefore the leg of a right-angled triangle, which is opposite an angle divided by the hypothenuse, gives the sine of that angle; and the leg of a right-angled triangle, adjacent to an angle divided by the hypothenuse, gives the co-sine of that angle. Therefore, when two sides of a triangle are given, the angles can be found.

PROB. XXXII.—Given two sides of a right-angled triangle, to find the angles by common arithmetic.

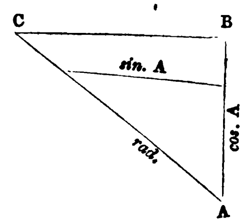
To do this by a table of natural sines, the hypothenuse must be one of the given parts, therefore if the two legs are given, the hypothenuse must be found by Prob. XXV; hence, when any two of a right-angled triangle are given, the angles may be found. We shall therefore suppose the hypothenuse always to be one of the given parts.

RULE.

If one of the given sides be the leg opposite the angle, divide this side by the hypothenuse, and the quotient will be the sine of the angle; and if one of the given sides be the leg adjacent to the angle, divide this side by the hypothenuse, and the quotient will be the co-sine of the angle, as has already been shown.

EXAMPLE I.

In the triangle $A\ B\ C$, are given the side $B\ C$ equal to 17·374 feet, and the hypothenuse $A\ C$ equal to 22·063 feet, to find the angle A. Here 17·374÷22·083=·78688.



By inspecting the table of natural sines, we find the two numbers 78676 and 78693, the first less than the sine of 51° 53′, and the second greater than the sine of 51° 54′; and since, in our calculations, a minute is of very little consequence, we shall not be far wrong in calling the quantity which the angle A contains 51° 53′.

EXAMPLE II.

Given the side $A\ B$ equal to 13·63 feet, and the hypothenuse $A\ C$ equal to 22·083, to find the angle A.

Here, 13·63÷22·083=·61721,

And in the table of co-sines, 61721 will be found to answer to 51° 53′.

EXAMPLES FOR PRACTICE.

In the right-angled triangle *A B C*, if the hypothenuse *A C* be the length of one of the faces of the arch, and the angle *A* the angle of obliquity, the opposite leg *B C* is the width of the arch-way, and the adjacent leg *A B* the distance of obliquity. See the construction of the plan of an oblique arch, page xlvi, and plate 12.

Therefore, when the width of the arch-way, and the length of one of the faces are given, the quotient obtained by dividing the width of the arch-way by the length of the given face, shall be the sine of the angle of obliquity.

And the quotient obtained by dividing the distance of obliquity by the length of the given face, shall be the co-sine of the angle of obliquity.

1. Given the width of the arch equal to 17·374 feet, and the length of one of the faces equal to 22·083 feet, to find the angle of obliquity? *Ans.* 51° 53′.

2. Given the distance of obliquity equal to 13·63 feet, and the length of one of the faces equal to 22·083 feet, to find the angle of obliquity? *Ans.* 51° 53′.

3. Given the width of the arch equal to 38·75 feet, and the length of one of the faces equal to 39·82 feet, to find the angle of obliquity? *Ans.* 76° 41′.

4. Given the distance of obliquity equal to 9 feet 2 inches, and the length of one of the faces equal to 39·82 feet, to find the angle of obliquity? *Ans.* 76° 41′.

5. Given the width of the arch-way equal to 45·96 feet, and the length of one of the faces equal to 60 feet, to find the angle of obliquity? *Ans.* 50°.

6. Given the distance of obliquity equal to 38·57 feet, and the length of one of the faces equal to 60 feet, to find the angle of obliquity? *Ans.* 50°.

7. Given the width of the arch-way equal to 19 feet, and the length of one of the faces equal to 42 feet, to find the angle of obliquity? *Ans.* 26° 54′.

8. Given the distance of obliquity equal to 37·4556 feet, and the length of one of the faces equal to 42, to find the angle of obliquity? *Ans.* 26° 54′.

9. Given the width of the arch-way equal to 22·833 feet, and the length of one of the faces equal to 27·003 feet, to find the angle of obliquity? *Ans.* 57° 44′.

10. Given the distance of obliquity equal to 14·416 feet, and the length of one of the faces equal to 27·003 feet, to find the angle of obliquity? *Ans.* 57° 44′.

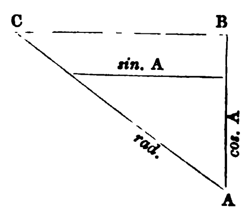
11. Given the width of the arch-way equal to 23 feet, and the length of one of the faces equal to 24·0416 feet, to find the angle of obliquity? *Ans.* 73° 4′.

12. Given the distance of obliquity equal to 7·009 feet, and the length of one of the faces equal to 24·0416 feet, to find the angle of obliquity? *Ans.* 73° 4′.

13. Given the width of the arch-way equal to 17 feet, and the length of one of the faces equal to 19·781 feet, to find the angle of obliquity? *Ans.* 59° 15′.

14. Given the distance of obliquity, 10·1138 feet, and the length of one of the faces equal to 19·781 feet, to find the angle of obliquity? *Ans.* 59° 15′.

PROB. XXXIII.—Given a side and an angle of a triangle, to find one of the two remaining sides.



Suppose the leg of the tabular triangle, which is called the sine, to be parallel to the leg of the proposed triangle, which is opposite the angle, and the hypothenuse of the tabular triangle to be upon the hypothenuse of the proposed triangle, and the leg adjacent to the angle to fall upon the leg of the proposed triangle; then in the tabular triangle, as the hypothenuse is called the radius, the leg opposite the angle the sine, and the leg adjacent to the angle the co-sine, the radius, sine, and co-sine, will form a triangle similar to the hypothenuse, the base, and the leg adjacent to the angle of the proposed triangle, as in the diagram.

EXAMPLE I.

Given the angle A equal to 26° 54′, and the side $A\ C$ equal to 42 feet, to find the side $B\ C$.

Rad. : sin. A :: $A\ C : B\ C$;

or $1 : {\cdot}45243 :: \ 42 \ : B\ C = {\cdot}45243 \times 42 = 19{\cdot}002$, or 19 feet.

EXAMPLE II.

Given the angle A as before, and the side $A\,B$ equal to 37·455 feet, to find the side $B\,C$.

Cos. A : sin. A :: $A\,B$: $B\,C$;

or $\cdot 8918 : \cdot 45243 :: 37\cdot 455 : B\,C = \frac{\cdot 45243 \times 37\cdot 455}{\cdot 8918} = 19\cdot 001$, or 19 feet nearly.

EXAMPLE III.

Given the angle A, as at first, and the side $A\,C$ equal to 42 feet, to find the side $A\,B$.

Rad. : cos. A :: $A\,C$: $A\,B$;

or $1 : \cdot 8918 :: 42 : A\,B = \cdot 8918 + 42 = 37\cdot 4556$ feet.

EXAMPLE IV.

Given the angle A, as at first, and the side $B\,C$ equal to 19 feet, to find the side $A\,B$.

Sin. A : cos. A :: $B\,C$: $A\,B$.

$\cdot 45243 : \cdot 8918 :: 19 : B\,B = \frac{\cdot 8918 \times 19}{\cdot 4524} = 37\cdot 454$ feet.

EXAMPLE V.

Given the angle A, and the side $A\,B$, equal to 37·455 feet, to find the side $A\,C$.

Cos. A : rad :: $A\,B$: $A\,C$.

$\cdot 8918 : 1 :: 37\cdot 455 : A\,C = \frac{37\cdot 455}{\cdot 8918} = 41\cdot 99$, or 42 feet nearly.

EXAMPLE VI.

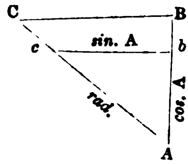
Given the angle A, as at first, and the side $B\,C$ equal to 19 feet, to find the side $A\,C$.

Sin. A : rad. :: $B\,C$: $A\,C$.

$\cdot 45243 : 1 :: 19 : A\,C = \frac{19}{\cdot 45243} = 41\cdot 995$, or 42 feet nearly.

EXAMPLES FOR PRACTICE.

In such of these examples where the distance between the faces and the angle of obliquity are given to find the length of the springing lines, the angle A of the triangle $A\ B\ C$ is equal to the angle of obliquity, the leg $B\ C$, opposite to A, is the distance between the two faces, and the hypothenuse $A\ C$ is the length of the springing lines, or length of each abutment. See figures 1, 2, 3, plate 12.



It will therefore be—

sin. A : cos. A :: $B\ C$: $A\ B$, the distance of obliquity,
sin. A : 1 :: $B\ C$: $A\ C$, the length of each face,
1 : sin. A :: $A\ C$: $B\ C$, the width of the arch-way,
sin. A : 1 :: $B\ C$: $A\ C$, the length of each abutment.

1. Given the angle of obliquity equal to 51° 53′, and the width of the arch-way equal to 17·34 feet, to find the distance of obliquity? *Ans.* 13·60 *feet.*

2. Given the angle of obliquity equal to 76° 42′, and the width of the arch-way equal to 38·75 feet, to find the distance of obliquity. *Ans.* 9·16 *feet.*

3. Given the angle of obliquity equal to 76° 42′, and the distance between the two faces equal to 26 feet, to find the lengths of the two springing lines or abutments. *Ans.* 26·71 *feet.*

4. Given the angle of obliquity equal to 50°, and the length of one of the faces equal to 60 feet, to find the width of the arch-way? *Ans.* 45·962 *feet.*

5. Given the angle of obliquity equal to 50°, and the length of one of the faces equal to 60 feet, to find the distance of obliquity? *Ans.* 38·567 *feet.*

6. Given the angle of obliquity equal to 50°, and the distance between the two faces of the arch equal to 28·75 feet, to find the length of the abutments? *Ans.* 37·53 *feet.*

7. Given the angle of obliquity equal to 26° 54′, and the length of one of the faces equal to 42 feet, to find the width of the arch-way? *Ans.* 19·002 *feet.*

8. Given the angle of obliquity equal to 26° 54′, and the length of one of the faces equal to 42 feet, to find the distance of obliquity? *Ans.* 37·4556 *feet.*

9. Given the angle of obliquity equal to 26° 54′, and the distance between the two faces of the arch equal to 14 feet, to find the length of the abutments? *Ans.* 30·944 *feet.*

10. Given the angle of obliquity equal to 57° 44′, and the length of one of the faces equal to 27·003 feet, to find the width of the arch-way? *Ans.* 22·833 *feet.*

11. Given the angle of obliquity equal to 57° 44′, and the length of one of the faces equal to 27·003 feet, to find the distance of obliquity? *Ans.* 14·416 *feet.*

12. Given the angle of obliquity equal to 57° 44′, and the distance between the two faces of the arch equal to 12·5 feet, to find the length of the abutments? *Ans.* 14·782 *feet.*

13. Given the angle of obliquity equal to 73° 4′, and the length of one of the faces equal to 24·0416 feet, to find the width of the arch-way? *Ans.* 22·999 *feet.*

14. Given the angle of obliquity equal to 73° 4′, and the length of one of the faces equal to 24·0416 feet, to find the distance of obliquity? *Ans.* 7·002356 *feet.*

15. Given the angle of obliquity equal to 73° 4′, and the distance between the two faces equal to 12·1666 feet, to find the length of the abutments or springing lines? *Ans.* 12·7191 *feet.*

16. Given the angle of obliquity equal to 59° 15′, and the length of one of the faces equal to 19·781 feet, to find the width of the arch-way? *Ans.* 16·9999 *feet.*

17. Given the angle of obliquity equal to 59° 15′, and the length of the oblique face equal to 19·781 feet, to find the distance of obliquity? *Ans.* 10·1138 *feet.*

18. Given the angle of obliquity equal to 59° 15′, and the length of each abutment equal to 30·875 feet, to find the distance between the two faces? *Ans.* 26·534 *feet.*

SECTION VII.

ON THE USE OF THE OBLIQUE-ANGLED TRIANGLE.

PROB. XXXIV.—To find a point in the side of a triangle, in which a perpendicular drawn from the opposite angle meets that side.

Let $B\,C=a$, $A\,C=b$, and $A\,B=c$; also let $B\,D=x$, and $C\,D=y$; then will $A\,D=c-x$.

By Euclid, Book I, Proposition XLVII:—

$$y^2=a^2-x^2$$

$$y^2=b^2-(c-x)^2$$

By equality,

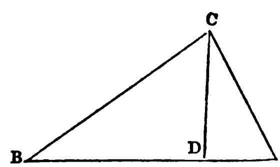
$$b^2-(c-x)^2=a^2-x^2$$

$$\text{Or } b^2-c^2+2cx-x^2=a^2-x^2$$

$$\text{Or } b^2-c^2+2cx=a^2\text{*}$$

$$2cx-c^2=a^2-b^2;$$

$$\text{Or } c(2x-c)=(a+b)(a-b)$$



Hence, by converting the equation into a proportion, we get

$$c : a+b :: a-b : 2x-c$$

Now, since $x=B\,D$, one of the segments of the base, and $c-x=A\,D$, the other segment, if x be the greater segment, $c-x$ is the less, and $c-x$ subtracted from x is $x-(c-x)=2x-c$, the difference of the segments of the base; therefore it will be—

As the base or longest side of a triangle, is to the sum of the other two sides; so is the difference of these two sides to the difference of the segments of the base.

By this rule, the distance of the perpendicular upon the base from one of its ends will be easily found; for, having found the difference of the bases of the two right-angled triangles, add half the difference to half the longest side $A\,B$, of the triangle $A\,B\,C$, the sum will be the length of $B\,D$, the greater segment; and subtract half the difference from half the base $A\,B$, the difference will be the length $A\,D$ of the less segment.

* Since $b^2-c^2+2cx=a^2$, by transposition, $2cx=a^2+c^2-b^2$, hence $x=\dfrac{a^2+c^2-b^2}{2c}$; but in the right-angled triangle $B\,D\,C$, right-angled at D, making $B\,C=a$ radius, then $B\,D=x$ shall be the co-sine of the angle B;

Hence radius$=1$: cos. B : : a : $x=a$ cos. B.

$$\text{Hence } a \cos. B=\frac{a^2+c^2-b^2}{2c}$$

$$\text{Hence } \cos. B=\frac{a^2+c^2-b^2}{2ac}$$

PROB. XXXV.—Having executed the piers of an oblique arch to the height of the springers, to draw the plan of the arch-way from the measures then taken.

EXAMPLE I.

Given the adjacent sides *H Q*, *H A*, (Fig. 1,) of the parallelogram *H A G Q*, and the diagonal *A Q*, respectively equal to 32ft. 6in., 22ft. 1in., and 25ft. 7½in., to draw a plan of the oblique arch, and to find the width of the aperture, *H A* being the length of one of the faces, and *H Q* the length of one of the abutments.

Draw the straight line *H Q*, and make *H Q* equal to 32ft. 6in.; from *H*, with the distance 22ft. 1in., describe an arc at *A*, and from *Q*, with the distance 25ft. 7½in., describe another arc cutting the former at *A*; join *A H*, *A Q*; draw *A G* parallel to *H Q*, and *Q G* parallel to *H A*; and the parallelogram *A H Q G* is the plan of the aperture. Draw *A C* perpendicular to *H Q*, meeting *H Q* in *C*, and *A C* is the width of the arch-way.

EXAMPLE II.

Given *A H* (Fig. 2,) equal to 19ft. 9⅞in., and *H Q* equal to 30ft. 10½in., and the perpendicular *A C*, which is the width of the arch, equal to 17ft., to draw the plan, *A H* being the length of one of the faces, and *H Q* the length of one of the abutments, and the angle *A H Q* being the angle of obliquity.

Draw the straight line *A H*, and make *A H* equal to 19ft. 9⅞in.; upon *A H*, as a diameter, describe the semi-circle *A C H*; from *A*, with the distance of 17ft., cut the semi-circular arc in *C*, and join *A C*, *C H*; prolong *H C* to *Q*, and make *H Q* equal to 30ft. 10½in.; draw *A G* parallel to *H Q*, and *Q G* parallel to *H A*; and the parallelogram *A H Q G* is the plan.

EXAMPLE III.

In the parallelogram *H A G Q*, (Fig. 3,) which represents the plan of the aperture of an oblique arch, the abutment *H Q* measures 35ft. 6in., the perpendicular distance between the abutments measures 28ft., and the distance of obliquity 9ft. 8in.; to draw the plan of the arch.

Draw the straight line *A C*, and make *A C* equal to 28ft.; draw *C H* perpendicular to *A C*, and make *C H* equal to 9ft. 8in.; prolong

PLATE 13.

Introduction

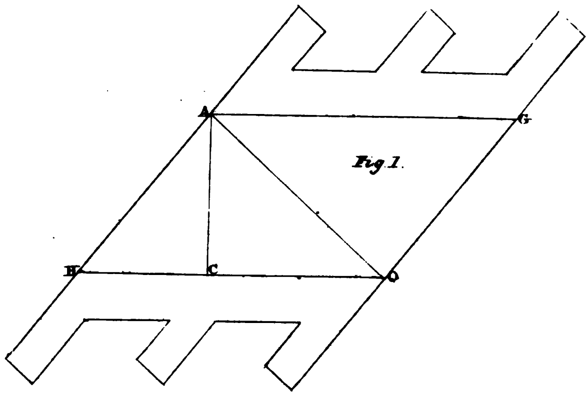


Fig. 1.

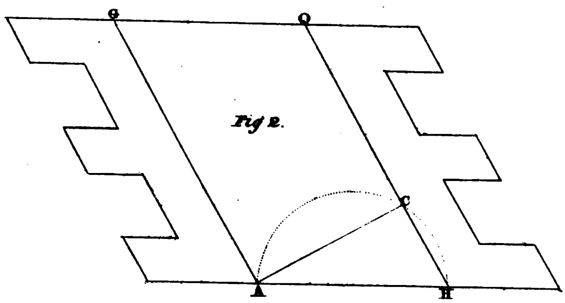


Fig. 2.

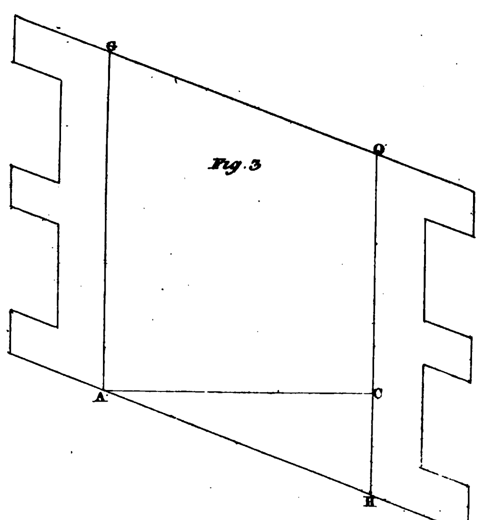


Fig. 3

$H\ C$ to Q, and make $H\ Q$ equal to 35ft. 6in.; join $A\ H$; draw $A\ G$ parallel to $H\ Q$, and $Q\ G$ parallel to $A\ H$; and $H\ A\ G\ Q$ is the plan of the aperture or arch-way required.

OBSERVATIONS.

The dimensions measured upon the side of the parallelogram which is to be one of the faces of the arch, and along one of the springing lines of one of the abutments, and the perpendicular distance between the abutment edges, as given in Example II, are the most convenient for construction, and require very little calculation. A plan drawn from the measures taken as in Example III, is also easily constructed, and requires no calculation; but the distance of obliquity must be previously found by a perpendicular. The dimensions taken as in Figure 1, are not only inconvenient with regard to the position of the plan, but require considerable calculation, as shown on the following

CALCULATION OF THE PARTS OF EXAMPLE I, FROM PROP. XXXIV.

The base $H\ Q$ of the triangle $H\ A\ Q$, being........32ft. 6 in.=32·5
One of the other two sides $Q\ A$........................25ft. $7\frac{1}{2}$in.=25·62
And the third side $H\ A$..................................22ft. 1 in.=22·083
The sum of these two sides is...........................47ft. $8\frac{1}{2}$in.=47·708
And their difference.................................... 3ft. $6\frac{1}{2}$in.=3·542

Hence, $32{\cdot}5 : 47{\cdot}708 :: 3{\cdot}542 : \dfrac{47{\cdot}708 \times 3{\cdot}542}{32{\cdot}5} = 5{\cdot}2$, which is the difference of the segments of the base, very nearly.

Now, half the base is the half of.........................32·5=16·25
And half the difference of the segments is the half of 5·2= 2·6
Hence the greater segment................................ =18·85
And the less... =13·65
And $A\ C = \sqrt{(A\ H^2 - H\ C^2)} = \sqrt{(22{\cdot}083^2 - 13{\cdot}65^2)} = 17{\cdot}35$ feet.

CALCULATION FOR $C\ H$, EXAMPLE II.

$C\ H = \sqrt{(A\ H^2 - A\ C^2)} = \sqrt{(14968{\cdot}515625)} = 122{\cdot}345$in. 10·195 feet.

SECTION VIII.

ON THE RADIUS OF CURVATURE.

PROB. XXXIV.—Given the radius of the surface of a cylinder, and the angle of inclination which the development of a spiral line on that surface makes with a line drawn on the development parallel to the axis of the cylinder, to find the radius of curvature of the spiral.

Draw the straight line *U V* (Fig. 1) to represent the axis of the cylinder. In *U V* take any point, *I*, and through *I* draw *G H* perpendicular to *U V*. Make *I G*, *I H*, each equal to the radius of the cylinder. Parallel to *U V*, through *G*, draw *Q R*, and through *H* draw *S T*; then *Q R T S* will be a plane passing through the axis of the cylinder. From the point *I*, as a centre, with the radius *I H* or *I G*, describe the semi-circle *H B G*. In the triangle *A D F*, draw *D F* parallel to *U V*, and *A D* perpendicular to *D F*. Take the height *D F* at pleasure, and make the angle *D F A* equal to the angle which the spiral line makes with a straight line on the surface of the cylinder, or with a line on the surface of the cylinder parallel to its axis. Draw *G L* parallel to *A F*, meeting *Q R* in *G*, and *S T* in *L*, intersecting *U V* in *J*. Draw *J K* perpendicular to *G L*, and make *J K* equal to *I H*, or *I G* the radius of the cylinder. Upon the semi-axis major *G L*, and with the semi-axis minor *J K*, describe the semi-ellipse *G K L*, which shall be a section of the cylinder parallel to *A F*, the development of the spiral line, and which shall have the same radius of curvature at *K*, the extremity of the axis-minor, which the spiral has at the same point. Now, in the triangle *A D F*, let $DF=a$, $AD=b$, and let the hypothenuse $AF=h$; then $h^2=a^2+b^2$; moreover, let the radius of the cylinder be denoted by r;

Then, by similar triangles, *D A F*, *I G J*;

$$AD : AF :: GI : GJ;$$

Or $b : h :: r : GJ=\dfrac{rh}{b}$, which is the length of the semi-axis major;

but from the property of the ellipse,

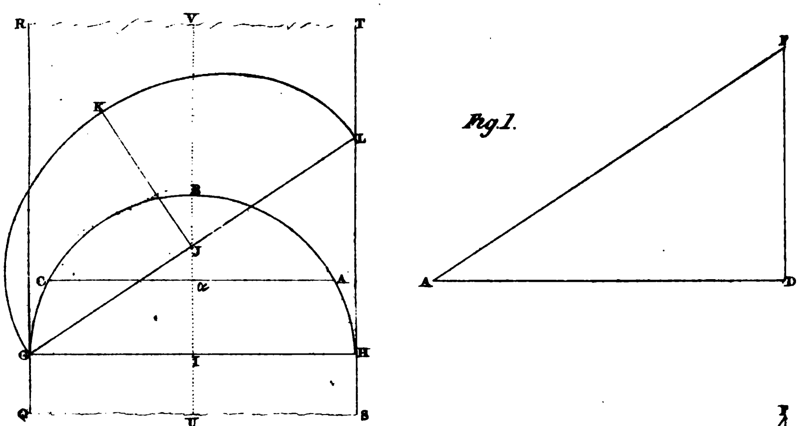


Fig.1.

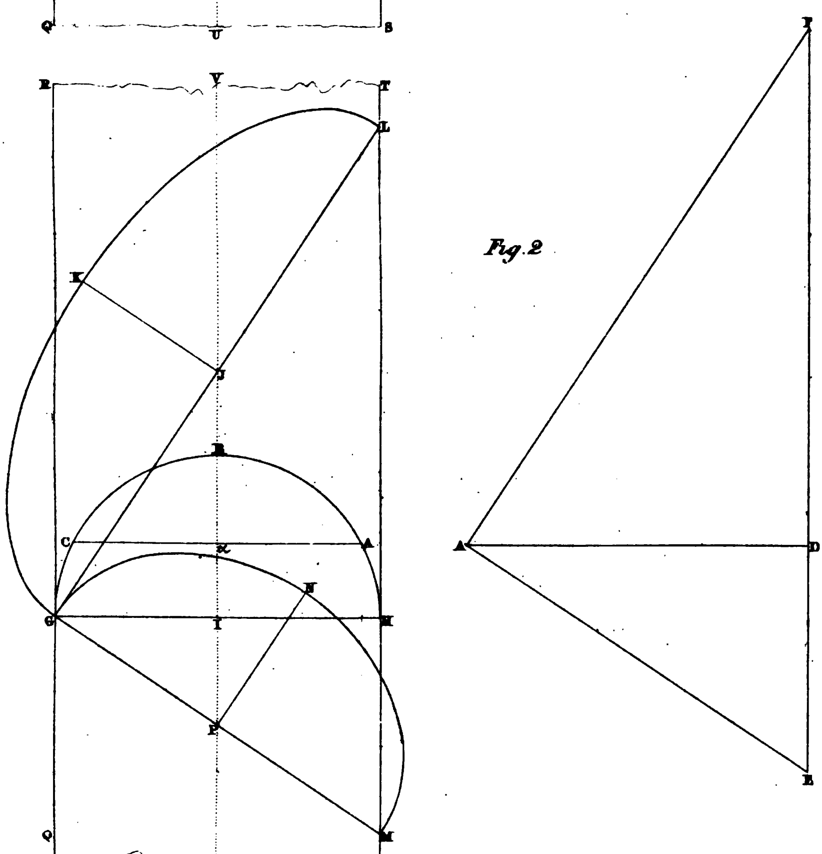


Fig.2.

$K J : G J :: G J$: the radius of curvature at K;

Or $r : \frac{rh}{b} :: \frac{rh}{b} : \frac{rh^2}{b^2} = \frac{r(a^2+b^2)}{b^2}$, which is the radius of curvature at K.

Now, the hypothenuse $A F$ of the triangle $A D F$, is the development or length of a spiral line, the base $A D = b$, a portion of the circumference of the cylinder, in a plane perpendicular to the axis, equal to the length of the arc $A B C$, passing from one extremity of the spiral, and $D F = a$ the length of a line on the curved surface of the cylinder, parallel to the axis drawn from the other extremity of the spiral, meeting the plane of the circle perpendicularly in the other extremity of the arc; hence we have the following

RULE.

Multiply the square of the length of the spiral line by the radius of the cylinder, divide the product by the square of the length of the arc, and the quotient is the radius of curvature.

EXAMPLE.

Given the radius of the well-hole of a spiral stair equal to 2 feet 3 inches, the height of a step equal to 6 inches, and the breadth of a step equal to 4 inches on the circumference, required the radius of curvature of the rail, and of the string-board under the steps.

The height of six steps is 3 feet, and the length of six steps round the arc upon which the cylindric surface of the well-hole stands is 2 feet; moreover, 2 feet 3 inches $= 2{\cdot}25$, the radius of the well-hole.

Now, $2^2 = 4$, the square of the length of the arc,
$3^2 = 9$, square of the height,
Sum $= 13$, square of the length of the spiral,
$13 \times 2{\cdot}25 = 29{\cdot}25$ prod. of rad. and square of length of spiral,
$29{\cdot}25 \div 4 = 7{\cdot}3125$ feet, the radius of curvature required.

Again, let $A B C$ (Fig. 2) be a section of the cylinder in a plane perpendicular to the axis, $A C$ being the chord, and $a B$ the height of the arc, and I the centre of the circle. Also, let $H G$ be drawn through I parallel to $A C$, and $U V$ through I perpendicular to $H G$, to represent the axis of the cylinder. Let $A D$ be drawn parallel to $A C$, making $A D$ equal to the length of the arc $A B C$. Draw $D E$ perpendicular to $A D$; make $D E$ equal to the distance of obliquity,

and join $A\,E$. Draw $A\,F$ perpendicular to $A\,E$, and prolong $E\,D$ to F. Then $A\,E$ will be the development or length of a spiral line in the face, parallel and equal to the lines which are the developments of the spirals of the joint-lines, and $A\,F$ will be parallel and equal to the lines which are the developments of the spirals of the bed-lines. Let the semi-circle $H\,A\,B\,C\,G$ be completed; draw $H\,M$ and $H\,L$ perpendicular to $H\,G$. Moreover, draw $G\,M$, $G\,L$, respectively parallel to $A\,E$, $A\,F$. Then $G\,M$ is the axis-major of an ellipse, of which the radius of curvature at the extremity of the axis-minor would be that of the spirals upon which the ends meet in the intrados; and $G\,L$ the axis-major of the ellipse, of which the radius of curvature at the extremity of the axis-minor would be that of the spiral bed-lines.

Then, because $E\,A\,F$ is a triangle right-angled at A, and $A\,D$ is drawn to meet the hypothenuse $E\,F$ perpendicularly in D, the triangle $E\,A\,F$ is divided into two right-angled triangles, which are similar to one another, and to the whole triangle $E\,A\,F$; therefore the sides about the equal angles are proportional, and the homologous sides are opposite equal angles.

Let $c=A\,D$, the length of the arc $A\,B\,C$, and $d=D\,E$, the distance of obliquity; also let $h=A\,E$; then $h^2=c^2+d^2$; moreover, let $r=P\,N=J\,K=I\,H$, the radius of the cylinder.

By similar triangles, $D\,A\,E$ and $I\,G\,P$,

$$D\,A : A\,E :: I\,G : G\,P;$$

Or $c : h :: r : G\,P=\dfrac{r\,h}{c}$, the semi-axis major of the ellipse $G\,N\,M$.

From the property of the ellipse and the radius of curvature,

$$P\,N : P\,G :: P\,G : \text{the radius of curvature at } N;$$

Or $r : \dfrac{rh}{c} :: \dfrac{rh}{c} : \dfrac{rh^2}{c^2}=\dfrac{r\,(c^2+d^2)}{c^2}$, which is the radius of curvature of the spiral lines of the joints, and is equal to that of the ellipse $M\,N\,G$, at the extremity N of the axis-minor.

Again, by similar triangles, $D\,E\,A$, or $D\,A\,F$, and $I\,G\,J$,

$$D\,E : E\,A :: I\,G : G\,J;$$

Or $d : h :: r : G\,J=\dfrac{rh}{d}$, the semi-axis major of the ellipse $G\,K\,L$;

and from the property of the ellipse, and the radius of the curvature,

$K J : J G :: J G :$ the radius of curvature at K;

Or $r : \dfrac{rh}{d} :: \dfrac{rh}{d} : \dfrac{rh^2}{d^2} = \dfrac{r(c^2+d^2)}{d^2}$, which is the radius of curvature of the ellipse $G K L$, at the extremity K of the axis-minor, and is equal to that of the spiral bed-lines.

RULE.

Divide the product of the radius of the cylinder, and the square of the length of the spiral joint-lines, by the square of the circular arc, and the quotient shall be the radius of curvature of the spiral joint-lines; and the same product divided by the square of the distance of obliquity, shall be the radius of curvature of the spiral bed-lines.

EXAMPLE.

Required the radius of curvature of each series of spiral lines for the edges of the bed-joints, given the chord equal to 38·75 feet, the height of the arc equal to 6 feet, and the distance of obliquity equal to $9\frac{1}{6}$ feet.

Since the chord is 38·75 feet, and the height of the arc 6 feet, the radius of the circle is 34·2825 feet. See the first of the Examples for Practice, Problem XXV, page xliii; and the length of the arc is 41·18156 feet, see Problem XXIV, Example II, page xli; but 34·2825 =34·28 nearly, and 41·18156=41·18 nearly.

$$A D^2 = 41{\cdot}18^2 = 1695{\cdot}7925$$
$$D E^2 = (9\tfrac{1}{6})^2 = 55^2 \div 6^2 = 84{\cdot}0277$$
$$A E^2 = A D^2 + D E^2 = 1779{\cdot}8202;$$

moreover $34{\cdot}28 \times 1779{\cdot}8202 = 61012{\cdot}236456$

$$\frac{61012{\cdot}236456}{84{\cdot}0277} = 729{\cdot}09 \text{ feet nearly;}$$

and $$\frac{61012{\cdot}236456}{1695{\cdot}7925} = 35{\cdot}97 \text{ feet.}$$

SECTION IX.

ON THE ANGLE OF THE TWIST.

PROB. XXXV.—To find the angle of the twist of two spiral lines in two given cylindric surfaces, and, in the same spiral surface, given the angle which the spiral in the inner cylindric surface makes, with a plane perpendicular to the axis of the cylinder.

Let *A B C* (Fig. 1) be a right section of the concave surface, the chord *A C* being 30 feet, and the height of the arc 10 feet; and let the concentric arc *D E F* be a right section of the convex surface which contains the other spiral, at 4 feet distant from the arc *A B C;* that is, whatever be the radius of the arc *A B C*, the radius of the arc *D E F* will be 4 feet more; let *g c a* be the angle which the spiral in the concave surface makes with its base, which is equal to the length of the arc *A B C;* and let the angle *a c g* of the triangle *g c a* be 40°.

The length of the arc *A B C* to a chord of 30 feet, and height 10 feet, will be found by Prob. XXIV, page xli, to be 38·22 feet; and the radius of the arc of the same dimensions, by Prob. XXV, page xliii, to be 16·25 feet; therefore 16·25+4·=20·25, the radius of the convex surface; and because the arcs of similar segments are as their radii; hence

16·25 : 20·25 :: 38·22 : the length of the arc *D E F*, which being found, is 47·628.

In the triangle *g a c* of the development of the spiral upon the concave surface, make *a c* equal to 38·22 feet, the angle *a c g* equal to 40°; then by the following statement,

rad. : tan. 40° :: 38·22 : *a g*=32·07 feet nearly.

In the right-angled triangle *g a f*, we have the two sides *a f*, *a g*, respectively equal to 47·628 feet, and 32·07 feet, to find the angle *a f g;*

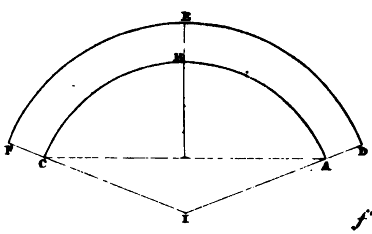
Therefore 47·628 : 32·07 :: rad. : tan. of the angle *a f g;*

Hence the angle *a f g* will be found to be about 33° 57′.

But without finding the lengths of the arcs, we might have arrived at the same conclusion by using the radii, which are proportional to

PLATE 15

Fig. 1



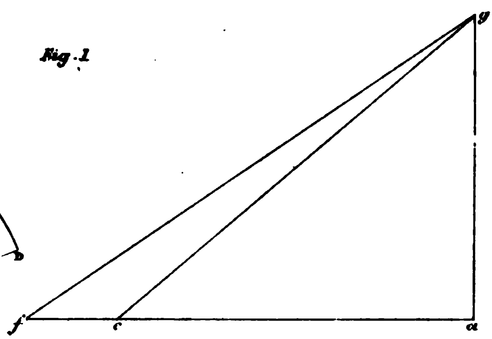
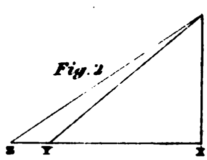
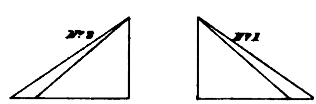


Fig. 2





the arcs. The radius $I\ A$ of the concave surface being found as above is 16·25 feet, and the radius $I\ D$ of the convex surface is 16·25+4=20·25 feet.

In the triangle $g\ a\ c$ we have the angle $a\ c\ g$=40°, and the side $a\ c$=16·25, to find the side $a\ g$, which by trigonometry will be found to be 13·63 feet.

Thus, rad. : tan. 40° :: 16·25 : ac=13·63.

And in the right-angled triangle $g\ a\ f$, we have the side $a\ f$=20·25, and the side $a\ g$=13·63, to find the angle $a\ f\ g$, which will be found to be 33° 57′, as before.

Thus, 20·25 : 13·63 :: rad. : tan. 33° 5′.

Now, $\angle\ a\ g\ f - \angle\ a\ g\ c = \angle\ a\ c\ g - \angle\ a\ f\ g$;

Or 40°—33° 57′=6° 3′, the angle of the twist.

TO FIND THE ANGLE OF THE TWIST IN PRACTICE.

It will be sufficient to divide the arcs $A\ B\ C$, $D\ E\ F$, each into the same number of equal parts, as eight, and the sum of the chords will be very nearly equal to the length of each respective arc. In the right-angled triangle $c\ a\ g$, make the base $a\ c$ equal to the whole of the eight parts of the arc $A\ B\ C$, or any number of them, and draw $a\ g$ perpendicular to $a\ c$. Draw $c\ g$, making the angle $a\ c\ g$ equal to 40°; prolong $a\ c$ to f, and make $a\ f$ equal to the whole of the eight parts of the arc $D\ E\ F$, or to the same number of them that $a\ c$ is of the arc $A\ B\ C$; draw $f\ g$; and the angle $c\ g\ f$ is the angle of the twist.

Or thus: Draw the straight line $X\ Z$ (Fig. 2), and $X\ W$ perpendicular to $X\ Z$. In $X\ Z$ make $X\ Y$ equal to $I\ A$ or $I\ C$, the radius of the concave surface; and make $X\ Z$ equal to $I\ D$ or $I\ F$, the radius of the convex surface; and make the angle $X\ Y\ W$ equal to 40°; join $Z\ W$; and the angle $Y\ W\ Z$ is the angle of the twist.

For the arcs of similar segments are as the radii. Thus, $I\ A : I\ D$: arc $A\ B\ C$: arc $D\ E\ F$.

If $A\ B\ C\ F\ E\ D$ be considered as the plan of a winding-stair, $a\ c$ the stretch-out of the ends of the steps round $A\ B\ C$, and $a\ f$ the stretch-out of the ends of the steps round $D\ E\ F$; and if the angle $a\ c\ g$ be the inclination of the spiral which passes through the nossings in the cylindric surface over the arc $A\ B\ C$, the angle $a\ f\ g$ shall be the inclination of the steps in the cylindric surface which stands upon the arc $D\ E\ F$. The angle $c\ g\ f$, which is the difference of the angles $a\ g\ f$ and $a\ g\ c$, shall be the angle of the twist which will be found necessary in working the soffit of the stair to a spiral

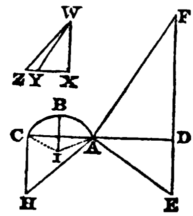
surface, observing that the ends of a single step are in the same proportion as the triangles *g a c*, *g a f*, and differ from a plane surface by an angle equal to *c g f*. If the inclination of the steps begin with the line *A D*, the stair is said to be right-handed.

This principle is the most eligible in constructing the spiral pump attributed to Archimedes.

In the step of a right-hand stair properly prepared, the workman will find, by placing himself adjacent to the concave end, the convex end to be sunk from the right to the left side of the stone in a straight line, so that this line will make an angle below the plane which touches the curve line of the upper edge of the concave end, equal to the angle of the twist.

And in the step of a left-hand stair properly prepared, the workman will find, by placing himself adjacent to the concave end, the convex end to be sunk from the left to the right side of the stone in a straight line, so that this line will make an angle below the plane which touches the curve line of the upper edge of the concave equal to the angle of the twist. Thus, No. 1 shows the ends of the step of a right-hand stair; No. 2 the ends of the step of a left-hand stair.

PROB. XXXVI.—In an oblique arch are given the chord of the right section of the concave surface equal to 20 feet, the height of the arc equal to 7 feet, the distance of obliquity equal to 16 feet, and the breadth of the beds equal to 18 inches, or 1ft. 6in.; to find the angle of the twist.



Draw the straight line $C\ D$, and in $C\ D$ make $A\ C$ equal to 20 feet, and upon $A\ C$ describe the arc of a circle, of which the height is 7 feet; make $A\ D$ equal to the length of the arc $A\ B\ C$; through C draw $C\ H$, and through D draw $F\ E$ perpendicular to $C\ D$; make $C\ H$ and $D\ E$ each equal to 16 feet, the distance of obliquity, and join $A\ H$ and $A\ E$; draw $A\ F$ perpendicular to $A\ E$; and the angle $D\ A\ F$ will then be equal to the angle $D\ E\ A$.

Draw $X\ Y$ parallel to $A\ D$, and make $X\ Y$ equal to $I\ A$ or $I\ C$, the radius of the arc $A\ B\ C$; draw $X\ W$ perpendicular to $X\ Y$, and draw $Y\ W$ parallel to $A\ F$; prolong $X\ Y$ to Z, and make $Y\ Z$ equal to 18in.; therefore $X\ Z$ will be equal to the radius of the convex surface. Join $Z\ W$, and the angle $Z\ W\ Y$ shall be equal to the angle of the twist; or, by calculation, thus:

From the chord equal to 20, and height of the arc equal to 7, will be found (Prob. XXIV, page xli) 25·9994=26 nearly, for the length of the arc $A\ B\ C$, and from the same dimensions will be found (Prob. XXV, page xliii) 10·642, the radius of the arc or of the concave surface. In the triangle $A\ D\ E$, right-angled at D, are given the side $A\ D$ equal to 26, and the side $D\ E$ equal to 16, from which will be found the side $A\ E=30{\cdot}52$ nearly.

Then to find the angle $D\ A\ E$ equal to the angle $A\ F\ D$, equal to the angle $Y\ W\ X$, we have, by trigonometry,

$$A\,E : D\,E :: \text{rad.} : \text{sin. } D\,A\,E = \frac{D\,E}{A\,E} = \frac{16}{30{\cdot}52} = 52424 = \text{sin. } 31^\circ$$

$37'$; therefore the angle $Y\ W\ X=31^\circ\ 37'$.

$$D\,E : A\,D :: A\,D : D\,F = \frac{A\,D^2}{D\,E} = \frac{26^2}{16} = 42{\cdot}25.$$

By similar triangles, $A\ D\ F$, $Y\ X\ W$,

$$A\,D : D\,F :: Y\,X : X\,W = \frac{D\,F \times Y\,X}{A\,D} = \frac{42{\cdot}25 \times 10{\cdot}642}{26} = 17{\cdot}293$$

$=17{\cdot}3$ nearly.

In the right-angled triangle $W\ X\ Z$, we have $X\ Z=10{\cdot}642+1{\cdot}5$

$=12 \cdot 142$; from hence the two sides $W X$, $Z X$, respectively equal to $17 \cdot 293$ and $12 \cdot 142$, will be found $Z W=21 \cdot 13$.

$$Z W : Z X :: \text{rad.} : \sin. Z W X = \frac{12 \cdot 142}{21 \cdot 13} = \cdot 5746 = \sin. 35^\circ\ 4';$$

hence $Y W Z = 35^\circ\ 4' - 31^\circ\ 37' = 3^\circ\ 27'$, the angle of the twist.

The same operation by logarithms, without finding $A E$ and $W Z$.

$$A D : D E :: \text{rad.} : \tan. D A E = \frac{\text{rad.} \times 16}{26}$$

Log. tan. $D A E$=log. rad.+log. 16—log. 26.

log. rad.............10

log. 16..............1·204120

log. 26..............1·414973

tan. $\angle\ D A E$ 9·789147=log. tan. 31° 36′.

But angle $D A E$=angle $A F D$=angle $Y W X$;

therefore the angle $Y W X=31^\circ\ 36'$

$D E : D A :: D A : D F = 26^2 \div 16$.

log. $D F$=2 log. 26—log. 16

log. 26=1·414973

twice log. 26=2·829946

log. 16=1·204120

log.DF=1·625826=42·25

$A D : D F :: Y X : X W$

26 : 42·25 :: 10·642 : $X W = 42 \cdot 25 \times 10 \cdot 642 \div 26$

log. $X W$=log. 42·25+log. 10·642—log. 26

$\left.\begin{array}{l}\text{log. } 42\cdot 25 = 1\cdot 625827\\ \text{log. } 10\cdot 642 = 1\cdot 026124\end{array}\right\}$ sum. 2·651951

log. 26..............................1·414973

log. $X W$..........................1·236978=log. 17·25

$W X : Z X$ :: rad. : tan. $Z W X$

tan. $Z W X$=rad.$\times Z X \div W X$

log. tan. $Z W X$=log. rad.+log. $Z X$—log. $W X$=log. rad. +log. 12·142—log. 17·25

log. rad................10

log. 12·142............ 1·084219

log. 17·25 .... 1·236978

log. tan. $Z W X$=9·847241=tan. 35° 7′.

Therefore the angle of the twist is $35^\circ\ 7' - 31^\circ\ 36' = 3^\circ\ 31'$.

A

TREATISE ON THE OBLIQUE ARCH.

ON THE OBLIQUE ARCH WITH SPIRAL JOINTS.

If a straight line move perpendicularly upon another straight line at rest, in a fixed plane, in such a manner that if the distance passed over upon the fixed line, by one extremity of the moving line, be proportional to the angle which the moving line makes with the plane, the other extremity will describe a curve on the surface of a cylinder, called a *cylindric spiral.*

From this definition of a spiral line, it is evident that the radius of the cylinder is equal to the length of the line which describes the spiral.

A surface, described by a straight line, moving perpendicularly upon another straight line at rest, in such a manner that the distance passed over upon the fixed line, by one extremity of the moving line, may be proportional to the angle which the describing line makes with a fixed plane, is called a *spiral surface,* and the fixed line is called the *axis of the spiral surface.*

If a spiral surface be cut by a cylindric surface, having the same axis as the spiral surface, the cylindric section of the spiral surface is a *spiral line.*

An oblique arch with spiral joints, is that in which the surfaces of the beds and the surfaces of the joints are both spiral surfaces.

If an oblique arch with spiral joints be executed according to the principles here established, and cut by a plane perpendicular to the axis of the cylinder, the section will exhibit a series of straight lines, dividing the arc of a circle into smaller arcs, and the lines being prolonged, would meet in the centre.

A right-hand oblique arch is that when, by approaching one of its elevations, the right-hand abutment advances before the left.

A left-hand oblique arch is that when, by approaching one of its elevations, the left-hand abutment advances before the right.

If the segment of a cylinder be cut by two parallel planes obliquely to the axis, and if the cylindric surface between the planes be developed, the curves, which are developments of the surface and the sections, will be identical, and will each be bisected by the straight line which joins each extremity, and the two straight lines will be parallel.

The projections of spiral lines, in the same cylindric surface which have parallel developments, are identical curve lines.

A *cylindric spiral* is a line of double curvature, and has all its parts equally inflected.

The development of a cylindric spiral is a straight line.

A straight line drawn through any given point on the surface of a cylinder, will be parallel to the axis.

A circle drawn through any given point in the surface of a cylinder, will be in a plane perpendicular to the axis.

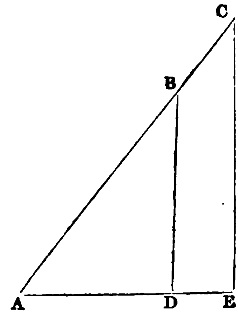
The development of the arc of a circle is a straight line, and is the length of that arc.

Therefore, if on the surface of a cylinder be given a spiral line, and if there be drawn on that surface a circle through one extremity of the line, and a straight line through the other; in the development of the cylindric surface, the three lines shall form a right-angled triangle, of which the development of the spiral shall be the hypothenuse, the length of the circular arc which may be called the base, one of the sides which contain the right angle, and the other side may be called the altitude of the spiral.

The angle of inclination of a spiral, is the angle which the development of the spiral makes with its base, or with the line which is the length of the circular arc.

The lengths of two spirals having equal inclinations, are to one another as their bases, or as their altitudes.

Let $A\ C$ be the development of a spiral, and let $C\ E$ be parallel, and $A\ E$ perpendicular to the axis of the cylinder. In $A\ C$ take any point B, and draw $B\ D$ parallel to $C\ E$, then the triangles $A\ E\ C$, $A\ D\ B$, are similar, and therefore have equal inclinations.



Hence, $A\,B : A\,D :: A\,C : A\,E$

Or $A\,B : A\,C :: A\,D : A\,E$

Again, $A\,B : B\,D :: A\,C : C\,E$

Or $A\,B : A\,C :: B\,D : C\,E$

Hence the lengths of two spirals having the same inclination, are to one another as their bases or altitudes.

If the bases and altitudes of two spiral lines have the same proportion, the two spirals have the same angle of inclination, and may form one continued spiral.

If the developments of two spiral lines, having equal altitudes but unequal bases, be bent upon two concentric cylindric surfaces, the least upon the concave, and the greater upon the convex surface, so as to have their proper altitudes, the two spiral lines shall be in a spiral surface, or will form the bed-lines of a course of stones; moreover the difference of the angles of inclination of the bed-lines is equal to the angle of the twist.

PROB. I.—To find the development and plan of the intrados of an oblique arch with spiral joints, given the width of the aperture, the height of the intrados, the angle of obliquity, the length of one of the springing-lines, and the number of arch-stones in each elevation.

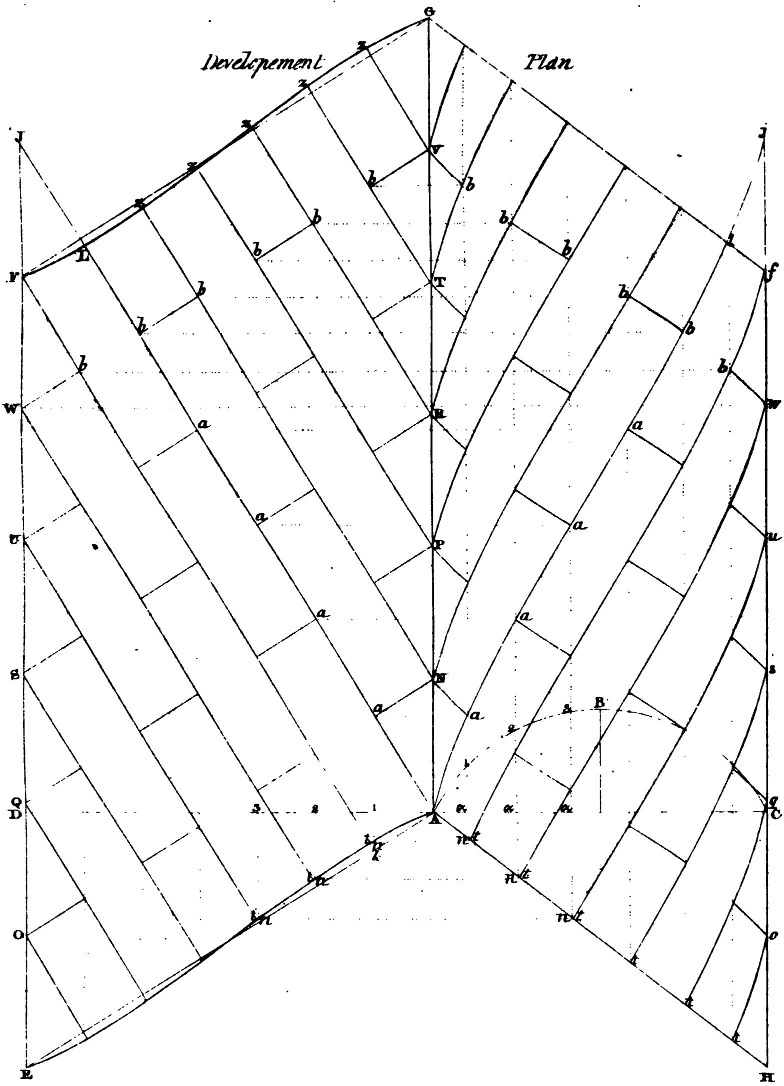
The method of the development of the curve of the oblique section of a cylinder has already been shown (Prob. XV, page xxii); but, for the sake of connection, the entire operation is here shown.

Draw *A C*, and make *A C* equal to the width of the aperture. Upon *A C* as a chord, with the height of the intrados, describe the arc *A B C*. Prolong *C A* to *D*, and make *A D* equal to the length of the arc *A B C*. Draw *A G* perpendicular to *C D*, and make *A G* equal to the length of the springing-line. Parallel to *A G*, through *D* draw *F E*, and through *C* draw *f H*. Make *D E* and *C H* each equal to the distance of obliquity. Join *A E* and *A H*, and complete the parallelograms *A H f G*, *A E F G*. *A H f G* is the outline of the plan, the lengths of the faces being *A H*, *G f*, and the lengths of the abutments or springing-lines *A G*, *H f*. The parallelogram *A E F G* is the outline of the development of the concave surface; except that instead of the straight lines *A E*, *G F*, there are also curves *A E*, *G F*, which are the developments of those portions of the intrados adjacent to each face of the arch.

Divide the arc *A B C* at the points 1, 2, 3, &c. and the straight line *A D* at the points 1, 2, 3, &c. each into an equal number of equal parts. From the points of division 1, 2, 3, &c. in the arc *A B C*, and from the points 1, 2, 3, &c. in the straight line *A D*, draw 1 *a n*, 2 *a n*, 3 *a n*, &c. 1 *n*, 2 *n*, 3 *n*, &c.; and let 1 *a n*, 2 *a n*, 3 *a n*, &c. meet the straight line *A H* in the points *n*, *n*, *n*, &c. and intersect the straight line *A C* in *a*, *a*, *a*, &c. Make the distances 1 *n*, 2 *n*, 3 *n*, &c. respectively equal to *a n*, *a n*, *a n*, &c. From *A* through the points *n*, *n*, *n*, &c. draw the curve line *A n n n*...*E*. With the edge of a thin slip of wood formed to the curve *A E*, draw the opposite identical curve *G F*, and the curves *A E* and *G F* are each bisected by the straight lines *A E*, *G F*, one-half of each curve being within, and the other half without the bisecting line, which we shall here call the regulating line.

Divide the straight line *A E*, which is the length of the face of the arch, into as many equal parts as there are arch-stones, which in the present example are seven, being designed for a small culvert. Through the points of division draw lines perpendicular to *A E*, and let one of these lines *k F*, meet the springing-line *E F* in the extremity *F*. Now, if lines were drawn through the points of division in *E k* parallel to *k F*, the springing-line *E F* would be divided into as many equal

PLATE 16



parts as the number of equal parts contained in *E k*, which is here six out of the seven, into which the whole line *A E* is divided.

As there are two methods of drawing the line *k L*, we shall suppose it drawn as here done, reserving the discussion of the mode which may be adopted for the conclusion.

Divide *A G* into six equal parts, at the points *N, P, R, T, V*, and draw *N O, P Q, R S, T U, V W*, parallel to *A E*, and *E F* will also be divided into six equal parts, at the points *O, Q, S, U, W*, which gives the lengths *E O, O Q, Q S*, &c. of the springers. From or through the points of division in *A E*, and from the springing-points *O, Q, S*, &c. *N, P, R*, &c. in *E F, A G*, draw lines parallel to *k F*, so as to meet the curve lines *A E, G F*, viz. *A E* in the points *t, t, t*, &c. and *G F* in the points *z, z, z*, &c. The parallel lines *t z, t z, t z*, &c. are the developments of the bed-lines, and the parallel lines *N O, P Q, R S*, &c. are the developments of the spirals, in which each series of joint-lines are parts. Perpendicular to *A G* draw *t t, t t, t t*, &c. meeting the straight line *A H* in the points *t, t, t*, &c. and the points *t, t, t*, &c. in the straight line *A H*, are the ends of the projections of the bed-line spirals.

Prolong *A L*, the development of one of the bed-line spirals, and the springing-line *E F*, to meet each other in *J*. Then, in order to construct the plan of the spirals of the bed and joint-lines, it will be sufficient to show the projection of one of each kind. Through the points 1, 2, 3, &c. in the arc *A B C*, parallel to *A G*, draw the straight lines *a a b, a a b, a a b*, &c. Let *a, a, a*, &c. be any number of points in *A L*, or in *A L J*. Perpendicular to *A G* draw *a a, a a, a a*, &c. meeting *a a b, a a b, a a b*, &c. in the points *a, a, a*, &c. From *A* through the points *a, a, a*, &c. draw the curve line *A a a a ... l j*, which is the projection of one of the bed-line spirals corresponding to the development *A a a a ... L J* of the same spiral. As the projection of spiral lines which have parallel developments are identical curve lines, and since we have the springing-points *N, P, R, T, V*, as also the ends *t, t*, &c. in *A H*, the remaining projections may be drawn, as before shown, by the edge of a thin slip of wood formed to the curve *A a a a ... l j*, observing that each end of the edge thus formed must be placed upon the springing-points in each springing-line, or at least one of the ends upon one of the springing-lines, and the edge successively upon the points *t, t, t*, &c. In the same manner may be found the curve line *V b b b...w*, which is the projection of the same joint-line, of which the straight line *V b b b... W* is the development. The remaining curves of the joint-lines may be drawn, as observed, in the curves of the bed-lines.

PROB. II.—Given the same things as in the preceding Problem, to find the elevation of one of the faces on a plane parallel to that face.

In drawing the plan or elevation of an oblique arch, the projections of all spirals which have parallel developments are identical curve lines, whether in a plan which is made on a plane parallel to the axis, or in an elevation which is made on a plane inclined to the axis. The projections on a plan differ considerably from those in an elevation.

In drawing the elevations of oblique arches, we must first find the projections of lines parallel to the axis of the cylinder passing through the points of division in the arc; then the projections of the spirals will be found by drawing straight lines, from the points in the projection of a spiral from the plan, to meet the lines which are parallel to the axis perpendicularly in the corresponding points of the projection of the spiral to be found, then drawing the curves through the points in the succeeding parallel lines.

Find the plan and development, as explained in the preceding Problem.

Draw the straight lines $p\ q$, $r\ s$, $t\ u$, $v\ w$, $x\ z$, &c. parallel to $A\ H$, making the distance between $p\ q$, $r\ s$, equal to the distance of the first joint below the springing-line, or about six inches, which is the usual allowance; also, make the distances of $t\ u$, $v\ w$, $x\ z$, &c. above the springing-line $r\ s$, respectively equal to n 1, n 2, n 3, &c. the heights of the ordinates of the arc $G\ K\ L$ above the chord $G\ L$. The lines $p\ q$, $r\ s$, $t\ u$, &c. are those which are parallel to the axis. Perpendicular to $A\ H$ draw $i\ i$, $j\ j$, $k\ k$, &c. meeting $r\ s$ in the points i, j, k, &c.; and i, j, k, are the projections of the bottoms of the indentations of the springers; also, perpendicular to $A\ H$, draw $l\ l$, $m\ m$, $n\ n$, &c. meeting $t\ u$, $v\ w$, $x\ z$, &c. respectively at the points l, m, n, &c. and draw the curve line $i\ l\ m\ n$, which is the projection of one of the courses of spiral joint-lines. In the same manner the entire elevation may be completed, as is evident from the figure.

In the projections of the spiral lines in the elevation, the plane of projection is not parallel to the axis of the cylindric surface, as is the case with the plane of projection upon which the plan is made. The projections of the spiral lines upon the plan are regular curves, but those in the elevation are not so uniform.

PROB. III.—The same things being given, to construct the longitudinal section.

This will be done in a similar manner to what has just been shown in the last Problem, as will be evident by inspecting Plate XVIII, and therefore any particular explanation will be unnecessary; observing that the curve line $r\ s\ t\ u$, No. 3, is the projection of the curve line $r\ s\ t\ u$, No. 1, and is therefore the spiral line, of which $r\ s\ t\ u$, No. 2, is the plan.

PLATE 17.

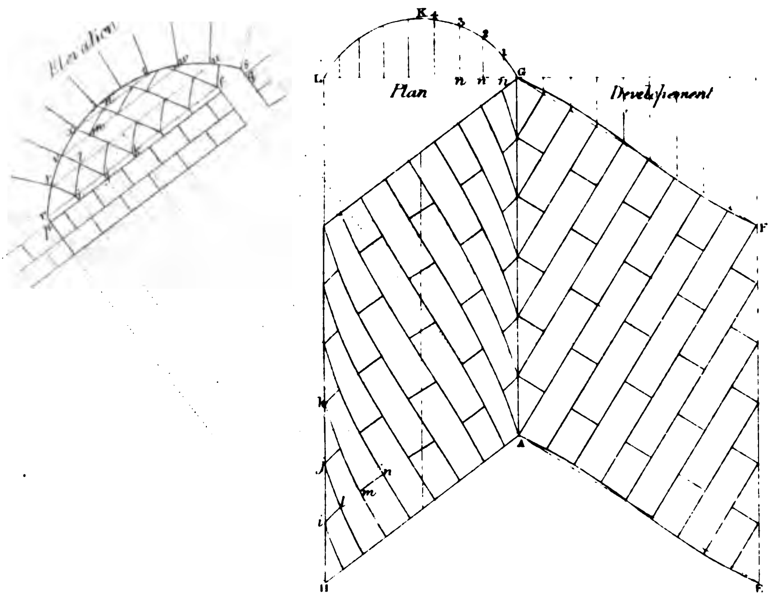


PLATE 19.

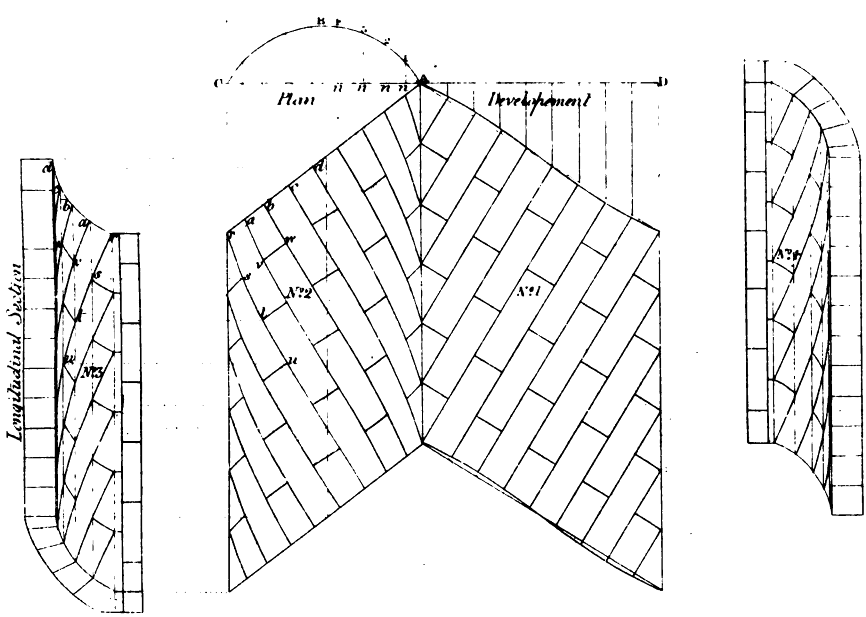
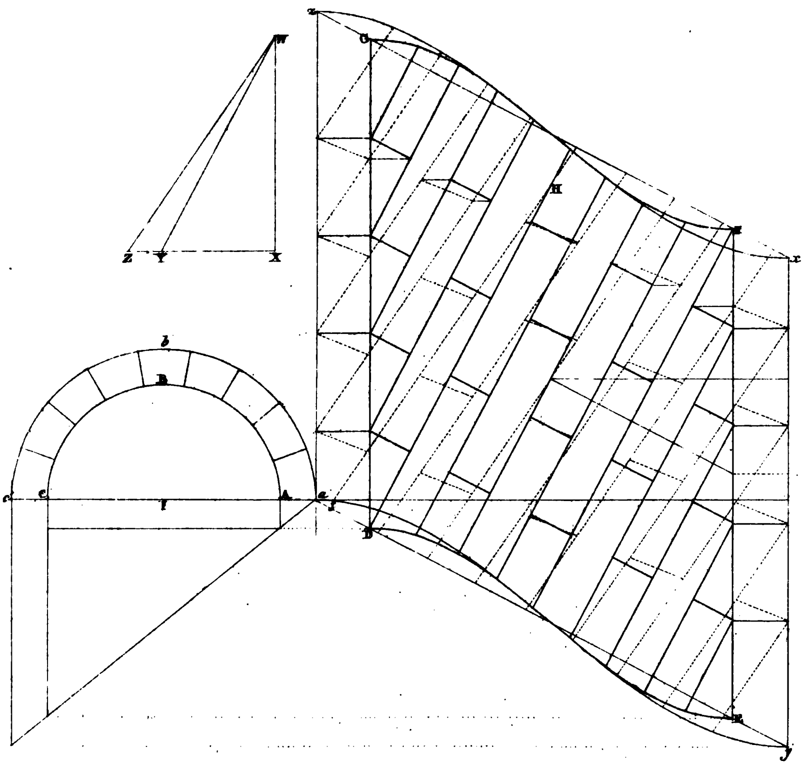


PLATE. 20



PROB. IV.—Having the length of the arc of the right section of the convex surface of an oblique arch, and the development of the concave surface, to find the development of the convex surface.

Let the semi-circle $a'\ b'\ c'$ be the right section of the convex surface, $A\ B\ C$ being that of the concave surface; and let $I\ E\ F\ G$ be the development of the concave surface constructed as in Prob. I, $I\ G$ and $E\ F$ being the springing-lines. The number of the springers in this example are five; therefore $I\ G$ is divided into five equal parts, at the points N, P, R, T, and $E\ F$ is also divided into five equal parts, at the points O, Q, S, U. The parallel lines $N\ O, P\ Q, R\ S, T\ U$, which are the developments of the joint-line spirals, are each divided by the developments of the bed-line spirals into as many equal parts as there are arch-stones in each elevation, viz. nine, as in this example. Therefore, divide the parallel lines $N\ O$, $P\ Q$, $R\ S$, $T\ U$, each into nine equal parts.

Prolong $A\ D$ to d. In $D\ d$, the part prolonged, make $a\ d$ equal to the length of the arc $a'\ b'\ c'$, and form the outline $a\ j\ x\ z$ in the same manner as $I\ E\ F\ G$, the development of the concave surface, $a\ z$, $j\ x$, being the springing-lines of this development. Parallel to $A\ D$ draw $N\ n$, $P\ p$, $R\ r$, $T\ t$, meeting $a\ z$ in n, p, r, t, as also parallel to $A\ d$ draw $O\ o$, $Q\ q$, $S\ s$, $U\ u$, meeting $j\ x$ in the points o, q, s, u. Join $n\ o$, $p\ q$, $r\ s$, $t\ u$, and the lines $n\ o$, $p\ q$, $r\ s$, $t\ u$, are the developments of the joint-lines in the convex surface, and are parallel to each other, and are also equi-distant. Divide the remote parallels $n\ o$, $t\ u$, each into nine equal parts; and as the lines $n\ o$, $t\ u$, comprise three steps or springers, the corresponding points through which the bed-lines pass will be three parts in advance from t further than from n. Therefore, the bed-line development $n\ y$, drawn from n in $n\ o$, will pass through the point 3 in $t\ u$, and will terminate in the curve $z\ x$ at y. Through each of the eight points of division in $n\ o$, draw lines parallel to $n\ y$ to meet the curve $z\ x$ and the curve $a\ j$. From the springing-points p, r, t, draw lines parallel to $n\ y$ to meet the curve $z\ x$, and from the springing-point o draw a line parallel to $n\ y$ to meet the curve $a\ j$, and we shall have the whole development of the convex surface as comprised by Fig. $a\ j\ x\ z$.

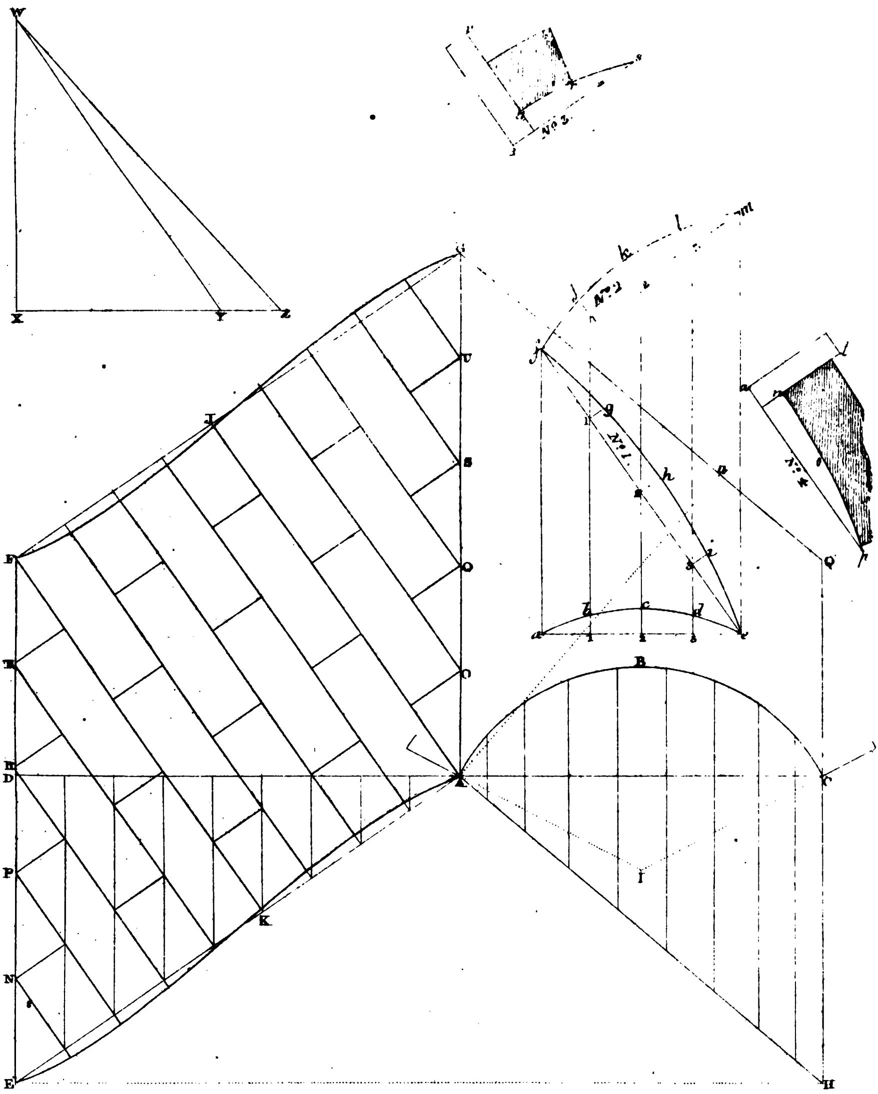
In Plate XX, the two developments are represented as if they were one diagram comprised by Fig. $a\ j\ x\ z$, in order to show that the angle of the twist is nothing more than the angle made by the developments of two bed-line spirals, viz. one in the concave, and the other in the convex surface.

In the right-angled triangle $W\ X\ Y$, draw $X\ Y$ parallel to the chord or diameter $A\ C$, $X\ W$ perpendicular to $X\ Y$. Make $X\ Y$ equal to the radius $A\ I$ of the concave surface, and draw $Y\ W$ parallel to $D\ H$, which is the development of one of the bed-line spirals. Prolong $X\ Y$ to Z, and make $Y\ Z$ equal to $A\ a'$. Join $Z\ W$, and $Z\ W$ shall be parallel to $J\ H$, the development of one of the bed-lines of the convex surface; therefore the angle $Y\ W\ Z$ is equal to the angle $D\ H\ J$.

PROB. V.—Given the chord and height of the intrados, the distance of obliquity, the distance between the two faces, the breadth of the beds, and the number of arch-stones, to construct the templets for working the arch-stones.

Draw *A C* equal to the chord, and with the height of the intrados describe the arc *A B C;* prolong *C A* to *D,* and make *A D* equal to the length of the arc. Draw *C H* and *D E* perpendicular to *C D;* and make *C H* and *D E* equal to the distance of obliquity. Join *A H, A E.* Describe the curve *A E* in the same manner as directed in Problem XV, page xxii, Introduction. Prolong *E D* to *F,* and *H C* to *Q′,* and draw *A G* parallel to *E F.* Perpendicular to *A H* draw *A u,* and make *A u* equal to the distance between the faces. Through *u* draw *G Q′* parallel to *A H,* and draw *G F* parallel to *A E.* Draw the curve line *G F,* and *A H Q′ G* is the outline of the plan, *A E F G* the outline of the development. In order to prevent two joints from meeting each other, it is necessary that the number of arch-stones in each face should be an odd number. We shall, therefore, suppose this number to be nine. Draw *F K* meeting the straight line *A E* perpendicularly in *K.* Divide *E K* into five equal parts, and *K A* into four equal parts, so that the whole line *E A* may consist of nine parts, as nearly equal to one another as the case will admit of. Now, there will be as many springers in *E F* as in the part *E K,* and as many in *A G* as in *E F;* therefore divide *E F* into five equal parts, at the points *N, P, R, T.* Draw *N O, P Q, R S, T U,* parallel to *E A,* meeting *A G* in *O, Q, S, U.* From the points *N, P, R, T,* and *O, Q, S, U,* as also from the points of division in *A K,* draw lines parallel to *K F* to meet the curve lines *A E, G F.* Then *K F,* and all the lines parallel to *K F,* are the developments of the bed-lines, and the alternate portions of the parallel lines *N O, P Q, R S,* &c. are the developments of the joint-lines. Draw *a e* parallel to *A C,* of any convenient length, and, with the radius *I A* or *I C,* describe the arc *a c e;* draw *a f* parallel to *A G,* and *e f* parallel to *K F.* Draw *e m* parallel to *A G,* and *f m* parallel to *E A.* In *a e* take any convenient number of points 1, 2, 3, &c. Draw the straight lines 1–1, 2–2, 3–3, &c. parallel to *A G,* intersecting the arc *a c e* in the points *b, c, d,* &c. and *f e* in the points 1, 2, 3, &c. and meeting *f m* in the points 1, 2, 3, &c. Perpendicular to *f e* draw 1 *g,* 2 *h,* 3 *i,* &c. and perpendicular to *f m* draw 1 *j,* 2 *k,* 3 *l,* &c. Make 1 *g,* 2 *h,* 3 *i,* &c. respectively equal to 1 *b,* 2 *c,* 3 *d,* &c. and likewise make 1 *j,* 2 *k,* 3 *l,* &c. respectively equal to 1 *b,* 2 *c,* 3 *d,* &c. Draw the curves *f g*

PLATE. 21



h i e, *f j k l m*. Make two templets (No. 1, No. 2) to form the segments *f h e*, *f k m*. Make also two arch-squares, (No. 3, No. 4,) having the inner edges of the one limb curved, and the other inner-edge straight, the straight edge being perpendicular to the under-edge of the curved limb, observing that the curved limbs, *q r s*, *n o p*, must be identical to the half-segments of No. 2, No. 1, viz. *q r s* the same as *k l m*, and *n o p* the same as *h i e*.

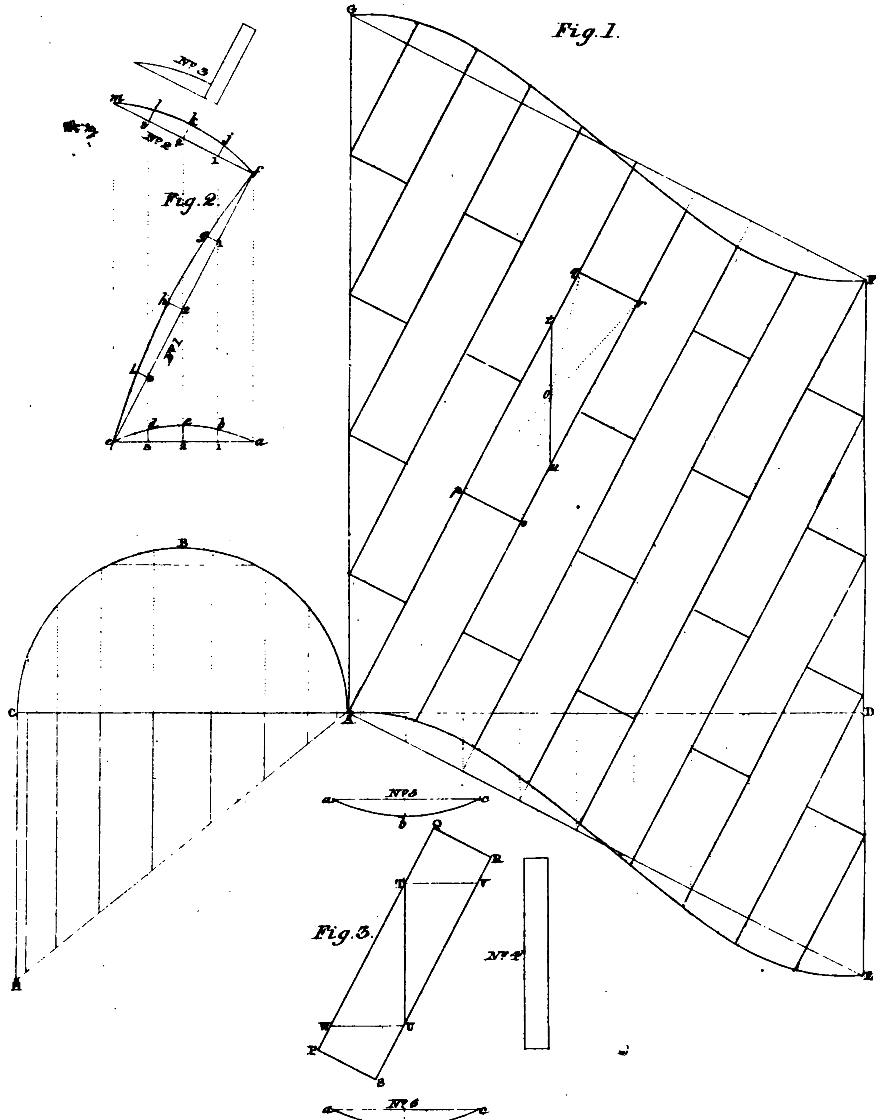
In the right-angled triangle *W X Y*, draw *X W* parallel to *A G*, and *X Y* parallel to *D A*; make *X Y* equal to the radius *I A* or *I C*, and draw *Y W* parallel to *K F*; prolong *X Y* to *Z*, and make *Y Z* equal to the breadth of the beds; join *Z W*; and the angle *Y W Z* is the angle of the twist.

TO WORK ONE OF THE ARCH-STONES.

Besides the templets already shown, prepare two straight edges, the one parallel, and the other broader at one end, containing the angle of the twist. We shall suppose that all the faces of the stone are rectangular, and that the stone is laid upon one of its beds, and consequently the other bed uppermost, and that you are placed adjacent to the soffit of the stone. Apply the templet No. 1, so that the curved edge *f g h i e* may rest upon each end of the arris next to you; then upon the surface of the stone draw a line by the curved edge, and the line thus drawn will be the bed-line between the soffit and that bed, observing that the curve must be convex to the opposite arris of the stone, or concave to the adjacent arris. Apply the parallel straight edge upon the arris next to you which is to be cut away, and the other straight edge which contains the angle of the twist, so that the planes of two faces of these straight edges may be parallel to each other, and at such a distance as to be equal to the intended breadth of the beds; then the straight edge which contains the angle of the twist must be sunk into the stone, until the upper edges of the two straight edges are out of winding, or are in one plane. The bottom of the chisel draught will then make an angle with the surface of the stone equal to the angle of the twist. If the arch is right-handed, the broader end of the straight edge must be applied next to the left hand of the stone; but if the arch is left-handed, the broader end must be applied next the right-hand end of the stone. The protuberant part between the bottom of the chisel draught and the bed-line must be taken away, until the surface agrees every where with a straight edge applied from any point perpendicular to the bed-line,

or to the curve which forms the bed-line. The spiral surface of the one bed being thus formed, we now proceed to form the soffit. Apply the arch-square, No. 3, in the same manner as a common square, each limb being perpendicular to the arris or to a tangent to the curve which forms the arris or bed-line, so that the straight edge *q v* may coincide with the surface of the bed, and the curved edge *q r s* may rest upon the stone which is attached to the soffit, and which is to be wrought away. Repeat the application until the curved edge *q r s* coincide with the surface of the soffit, which will then be that of a cylinder. Having formed the cylindric surface, guage the soffit to its breadth, and proceed with the same arch-square to work the other bed. It need hardly be observed that the curved edge must be applied to the soffit or cylindric surface, and, consequently the straight edge upon the winding surface of the bed.

PLATE. 22



ANOTHER METHOD OF FORMING THE ARCH-STONES.

The development, Fig. 1, and the templets, No. 1, No. 2, and No. 3, Fig. 2, being constructed, as explained in the last Problem.

Let *p q r s* be the development of the soffit of one of the stones. Find the centre *o*, by drawing the two diagonals *q s*, *r p*. Through *o* draw *t u* parallel to *A G*, meeting *p q* in *t*, and *s r* in *u*, and *t u* is a line parallel to the axis of the cylinder. Let *P Q R S* (Fig. 3) represent that face of the stone which is to be the soffit. With a straight edge, No. 4, cut a chisel draught, *T U*, through the centre of this face, making the angle *P T U* equal or nearly equal to the angle *p t u*, which the bed-lines make with the axis of the cylinder, so that the bottom of the chisel draught may be parallel to the face of the stone, and of sufficient depth to allow for forming the concave surface of the cylinder to the full extent. Make two templets, *a b c*, *a b c*, No. 5, No. 6, equal portions of the arc *A B C*, and let the points *b*, *b*, be the middle points of the circular edges. Perpendicular to *T U* draw *T V*, *U W*. Cut two chisel draughts under *T V*, *U W*, in such a manner that when the circular edges of the two templets, No. 5 and No. 6, are applied upon each of these draughts, and the middle points *b*, *b*, upon *T* and *U*, and if the straight edges *a c*, *a c*, are out of winding, and parallel to the surface of the stone, and if the bottoms of the draughts are sunk to the depth of the straight draught, and if the circular edges of the templets coincide with the bottoms of the two draughts, the bottoms of these two draughts shall be portions of the cylindric surface, or of the surface of the soffit to be formed. By means of the curved edge *e i h g f* of the templet No. 1, run two draughts along each margin of the stone adjacent to the bed-lines, to the same depth as the bottoms of the straight and circular draughts; and by the templet No. 2, run two draughts along the ends adjacent to the joint-lines, to the same depth as the other two. The superfluous stone between the draughts being cut away, will form the cylindric surface of the soffit.

Having made a limber mould* to the development *p q r s*, and having drawn the line *t u* upon this mould, press the mould upon the concave surface thus formed for the soffit, so that all the points may be in contact with the cylindric surface, and that the line *tu* upon the mould may fall

* Zinc is well adapted to this purpose.

upon *T U* on the cylindric surface of the stone. In this state draw four lines round the four edges of this mould upon the concave surface of the stone. The two longest of these lines will be the bed-lines, and the two shortest the joint-lines. The beds of the stone are formed by means of the arch-square No. 3, by applying the circular edge to the soffit. The other arch-square for forming the joints is not exhibited.

The reason of the construction and application of these moulds explained as above, as also in Prob. VI, page 15, will be made more evident by examining the explanation to Prob. VII, page 16.

II.

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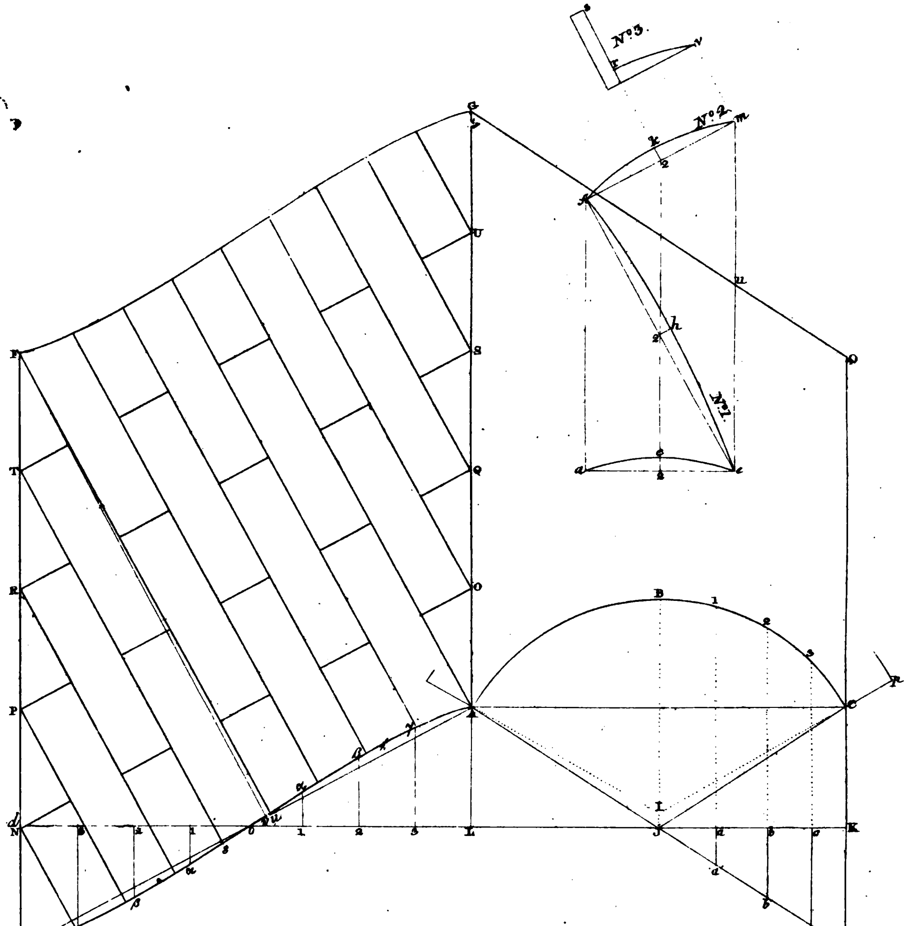
PLATE. 23



A SECOND EXAMPLE,

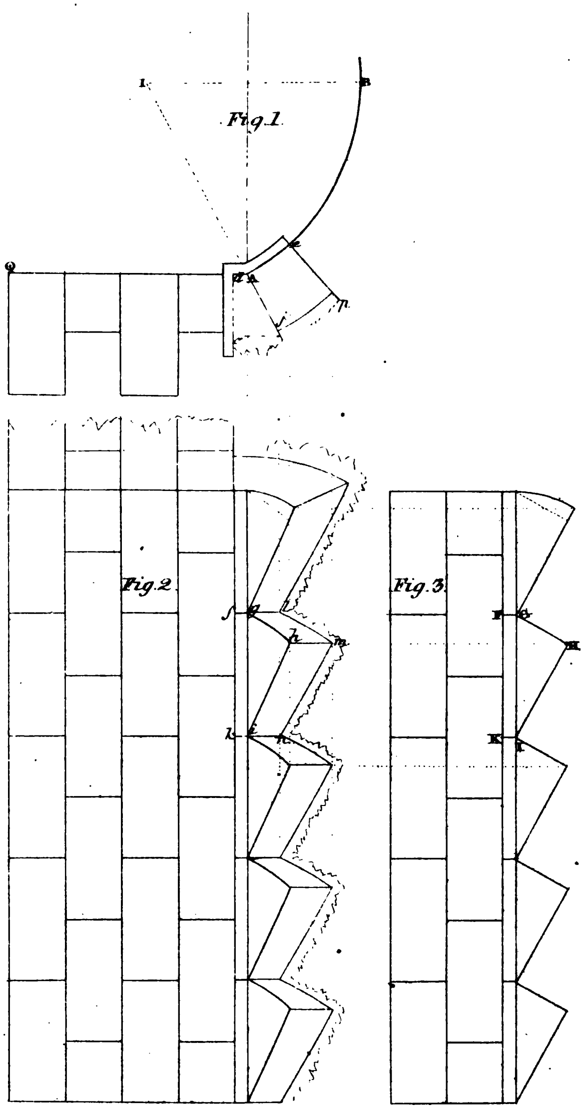
With a different method of finding the development of the curve, which is the intersection of the intrados, and the plane of the face of the arch, as also the method of finding the position of the bed-lines, so that the arch-stones may be equally thick.

Let *A H Q G* be the plan of the arch, *A H*, *G Q*, being the representation of the faces; and let the arc *A B C* be the right section, *A C*, the chord, being perpendicular to *A G*. Divide the arc *A B C* into two equal parts at *B*, and draw *B J* perpendicular to *A C*, meeting *A H* in *J*. Through *J* draw *d K* parallel to *C A*, meeting *H Q* in *K*. Prolong *G A* to meet *d K* in *L*, and make *L d* equal to the length of the arc *A B C*. Through *d* draw *E F* parallel to *A G*. Make *d E* equal to *K H*, and *E F* equal to *A G*. Join *A E*, and draw *G F* parallel to *A E*. Divide the arc *B C* into any number of equal parts, as here into four, and through the points 1, 2, 3, &c. draw lines parallel to *B J*, meeting *J H* at a', b', c', &c. and intersecting *J K* at *a*, *b*, *c*, &c. Bisect *L d* in *o*. Divide *o L*, *o d*, each into the same number of equal parts into which the arc *B C* is divided, viz. each into four, at 1, 2, 3, &c. Draw 1 α, 2 β, 3 γ, &c. on each side of the middle point *o*, and make 1 α, 2 β, 3 γ, &c. respectively equal to $a\,a'$, $b\,b'$, $c\,c'$, &c. From the point *o*, and through the points α, β, γ, &c. on each side of it draw the curves $o\ \alpha\ \beta\ \gamma \ldots A$, $o\ \alpha\ \beta\ \gamma \ldots E$, and the whole curve *A E* is the development of the cylindric surface, and the plane of the face of the arch. Draw the curve line *G F* so as to be identical to the curve line *A E*. Divide the straight line *E A* into as many equal parts as there are ring-stones in the face of the arch, as here into nine, and let *s*, *u*, be respectively the fourth and fifth points of division from *E*. Draw *F v* perpendicular to *A E*, meeting it in *v*; and as the point *v* falls between the fourth and fifth points, but nearer to the fifth *u* than to the fourth *s*, therefore join *F u*. Divide each springing-line *E F*, *A G*, therefore, into five equal parts, viz. *E F* at the points *N*, *P*, *R*, *T*, and *A G* at *O*, *Q*, *S*, *U*, and join *N O*, *P Q*, *R S*, *T U*. Through the points *N*, *P*, *R*, *T*, draw lines parallel to *F u* to meet the curve line *E A*; through the points *O*, *Q*, *S*, *U*, draw lines also parallel to *F u* to meet the curve line *G F*; and through the intermediate points of division in *u A*, still parallel to *F u*, draw lines to meet the curves *A E*, *G F*; then *F u*, and all the lines parallel to *F u*, are the developments of the

bed-lines, and the alternate portions of the lines *N O*, *P Q*, &c. are the developments of the joint-lines.

The templets No. 1, No. 2, No. 3, might have been drawn as before explained; or having drawn *a e*, *e f*, *f m*, and the arc *a c e*, as in Prob. V, bisect the straight lines *a e*, *e f*, *f m*. Through the point of bisection in *a e*, draw an ordinate; and from the points of bisection in the straight lines *e f*, *f m*, draw perpendiculars to the lines. Make the height of each of these perpendiculars equal to the ordinate upon 2 *a*; then by Prob. VII, page xii, Introduction, through the three points *e*, *h*, *f*, describe the arc *e h f*; and through the three points *f*, *k*, *m*, describe the arc *f k m*. These arcs do not practically differ from the curves found in Prob. V, which curves are portions of ellipses.

PLATE 24



PROB. VI.—To form the springers of an oblique arch.

Let *Q A B* (Fig 1) be half the right section of the arch, as shown in Plate XXIII, *A Q* being a section of the face of the abutment, *A B* the section of half the intrados, *c d A e p j* a section of one of the springers, the springers being set upon a level bed, generally six inches below the springing-line of the soffit or intrados. The bed-line upon which they are placed is represented in section by *c d*. Fig. 2 represents the inside face of the abutment as supporting the springers, *f g h i k* exhibits the concave face of one of the springers corresponding to *F G H I K* (Fig 3), *f g i k* being the part which is in the same plane with the face of the abutment, and the triangular surface *g h i* part of the soffit or intrados, each being one-fourth part of the development of one of the stones exhibited in Plate XXIII, where the springers are also exhibited upon the springing-lines *A G, E F*.

The concave surface *g h i* will be ascertained by the crooked templet *c d A e*, of which *c d* is a section of the bed, *d A* a section of the plane surface of the face of the plane part of the springer, and *A e* a portion of the right section of the intrados. This templet is used by applying the straight edge *c d* upon the bed marked *f k*, (Fig. 2,) which, from the nature of orthographical projection, cannot be seen as a surface, but as a line only. This will easily be understood by the workman. While applying the straight edge *c d* of the templet upon the bed, the straight part *d A* must be applied upon the narrow face *f g i k*, and the convex edge *A e* upon the concave surface *g h i*, which being formed, the bed-line *h i*, and the joint-line *h g*, must be drawn by bending a limber mould made to the triangle *G H I*, (Fig 3,) which is half the development of half of one of the stones exhibited in Plate XXIII.

The triangular soffit of the springers being thus formed, we now proceed with the upper bed, as represented by the quadrilateral figure *h m n i* (Fig. 2), and the end or abutting joint by Fig. *g h m l*.

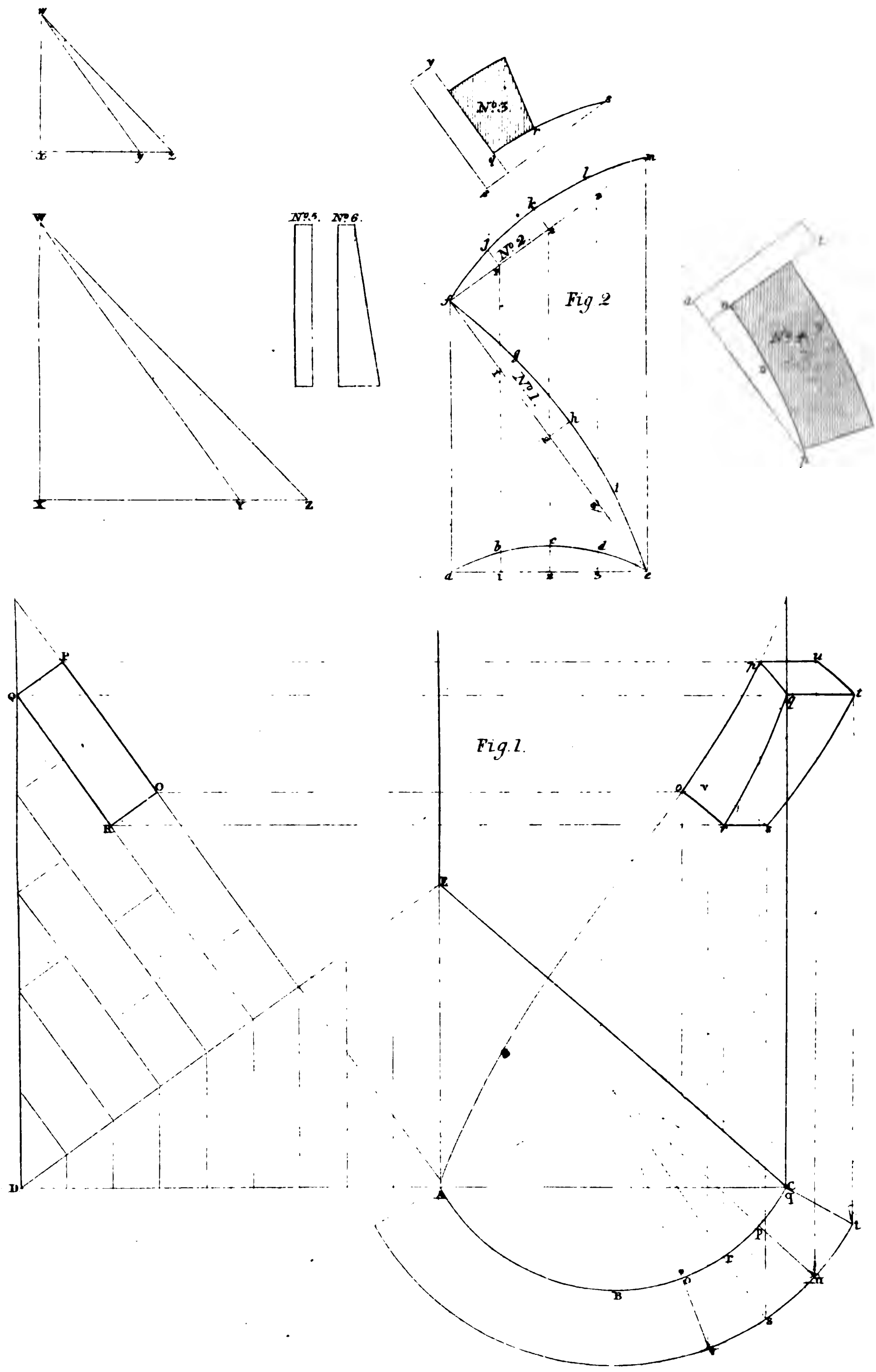
We may now suppose that the templets have been constructed as in Plate XXIII, and that the arch-square No. 3 is that for working the beds when the soffit is given or finished, or for working the soffits when the beds are given or finished. Then if the bed *h m n i* be so formed, that while applying the arch-square No. 3, so that the point *r* between the curved and straight edges may be upon the line *h i*, the curved edge *v r* upon the soffit, and the straight edge *r s* upon the upper bed, and the plane containing the two branches perpendicular to the bed-line *h i*, and if the straight edge coincide with the surface, the upper bed will be formed as required.

PROB. VII.—Having the projection or plan of a stone, to explain the nature of the cylindric surface of the intrados, the properties of the spiral surfaces of the beds and joints.

Let *O P Q R* (Fig 1) be the development of the intrados or soffit of a stone, *o p q r s t u v* the entire projection of the stone, *o p q r* representing the intrados, *u v s t* the extrados, *q r s t* the upper bed, *p o v u* the lower bed, *p q t u* the end of the stone which forms one side of the joint, the other side being formed by the end of the adjacent stone, which must be supposed to be previously set, and *r o v s* the side of the joint against which the end of the next stone is to come. Also, let No. 1, No. 2, No. 3, No. 4, (Fig. 2,) be the templets for working the stone, found as explained in Prob. V, page 8, and the angle of the twist *Y W Z*, found as explained at the top of page 9; observing that if there is a want of room, the operation may be condensed so as to occupy less space, by using equal portions of the radius, and the breadth of the beds as here, by making *x y* half the radius, and *y z* half the breadth of the bed; then the angle *y w z* shall be equal to the angle *Y W Z*. The templet No. 5 is a straight edge of equal breadth, and No. 6 a straight edge with tapering edges inclined to each other, making an angle equal to the angle of the twist.

With regard to the curvature of the different faces of the stone, all the sections of the intrados represented by *o p q r*, made by planes through the axis of the cylinder, are straight lines; and all the sections made by planes perpendicular to the axis, are equal arcs having the same radius as that of the cylinder, or that of the right section. In every other direction the section made by a plane passing through a straight line perpendicular to the axis, is a concave curve or a portion of an ellipse, at the extremity of the semi-axis minor, which is equal to the radius of the cylinder. The templet or convex rule No. 1 shall coincide with the convex surface *o p q r*, along the marginal or bed-lines *o p*, *r q*, and in every direction between and parallel to these lines; and the templet or convex rule No. 2 shall coincide with the marginal-lines *p q*, *o r*, and with all lines parallel to *p q*, *o r*.

In the side *q r s t*, which is the representation of the upper bed of the stone, there can be no sections of the surface made by a plane that are absolutely straight lines, excepting those in which the cutting-planes are perpendicular to the axis, as the lines *q t*, *r s*. The bed-lines *r q*, *s t*, are very nearly straight, being curves of contrary


No. 3.
No. 2
No. 1.
No. 4
No. 5.
No. 6.
Fig. 2
Fig. 1.
W
X
Y
Z
P
Q
O
R
D
E
A
B
C

flexure, the radius of curvature in the middle being infinitely great; therefore the tangent will very nearly coincide with the curve (see the sections, Fig. 1, Plate X, and explanation, page xxviii, Introduction.) It is upon this principle, however, that the stonecutter is enabled to work the bed independent of the soffit; as the bed of the stone is not only straight in lines perpendicular to the axis, but also nearly straight in lines made by the sections of planes which are parallel to the axis, so that, practically speaking, two straight lines may be drawn through any given point in the spiral bed perpendicular to each other, the one being perpendicular to the axis.

Because the arch under consideration is right-handed, the two lines *r q*, *r s*, being considered as in the same plane, it will require an angle to be made with a straight line *s t*, at the point *s* equal to the angle *Y W Z*, or *y w z*, in order to raise the line *s t* to the plane *s r q*, or the point *t* as much as the broader end of the tapering straight edge No. 6 exceeds the narrower end. Therefore, if the spiral bed be cut by a plane parallel to a plane between the lines *r s*, *r q*, the section of the surface shall be convex; and if cut by a plane parallel to a plane between the lines *q r*, *q t*, the section shall be concave; hence a section of the bed made by a plane passing through any point in the bed-line *r q*, parallel to the sectional line *E C* of the face of the arch, shall be concave. It is on this account that the joints between the ring-stones of the face of an oblique arch, are concave to the axis-minor of the ellipse.

The arch-square *v q r s*, No. 3, is applied to every point of the arris *r q*, as shown to work the soffit of the stone when the bed is previously wrought, or the bed of the stone when the soffit is wrought; and the arch-square *t n o p*, No. 4, is applied to work the ends of the stone as shown, the soffit and the beds being wrought, the two limbs of each arch-square being in a plane perpendicular to the arris.

PROB. VIII.—Supposing the breadth of the arch-stones to be cut into two rings of equal thickness, and the external removed, to find the development of the intrados of the inner ring, so that the development of the extrados may have the bed and joint-lines at right angles to each other, and to find the templets for ascertaining the form of the stones.

Let the right section of the intrados be the arc $A\ B\ C$, and let the arc $a'\ b'\ c'$ be the section which divides the arch-stones into two equal thicknesses. Prolong $C\ A$ to D, and make $A\ D$ equal to the length of the arc $A\ B\ C$. Draw $A\ G$ perpendicular to $A\ D$, and make $A\ G$ equal to the length of the abutment. Through D draw $E\ F$ parallel to $A\ G$. Make $D\ E$ equal to $C\ H$, and $E\ F$ equal to $A\ G$. Join $A\ E$ and $G\ F$, and draw the curve lines $A\ E$ and $G\ F$, as in the former examples. Bisect $A\ D$ in k, and draw $k\ g$ perpendicular to $A\ D$, meeting the straight line $A\ E$ in g. Prolong $k\ D$ to d, and make $k\ d$ equal to half the length of the arc $a'\ b'\ c'$. Through d draw $e\ f$ parallel to $E\ F$, and $g\ h$ perpendicular to $A\ E$, meeting $e\ f$ in h. Draw $h\ i$ perpendicular to $E\ F$, meeting $E\ F$ in i. Prolong $i\ h$ to j. Make $h\ j$ equal to $h\ i$, and join $i\ g$. Prolong $A\ E$ to e, and $F\ E$ to l. Draw $e\ l$ perpendicular to $F\ l$, and join $g\ l$. Divide the straight line $A\ E$ into as many equal parts as there are ring-stones, suppose nine; therefore divide $A\ E$ into nine equal parts. Draw $F\ K$ parallel to $i\ g$, and let the point K fall upon one of the points of division. Supposing that $K\ E$ contains five of the nine equal parts, therefore each abutment $A\ G$, $E\ F$, will have five springers. Divide $A\ G$, $E\ F$, each into five equal parts, viz. $A\ G$ at the points O, Q, S, U, and $E\ F$ at the points N, P, R, T. Through the points A, O, Q, S, U, draw lines parallel to $K\ F$ to meet the curve line $G\ F$; through the points N, P, R, T, draw lines also parallel to $K\ F$ to meet the curve line $A\ E$; and through the points of division in $A\ K$, draw line s again parallel to $K\ F$ to meet the curve line $G\ F$; and $K\ F$, as also all the lines which are parallel to $K\ F$, are the developments of the bed-lines upon the intrados or soffit of the arch. If the lines $N\ O$, $P\ Q$, $R\ S$, be drawn, they will be parallel to $E\ A$. We will suppose them to be drawn. Bisect the parts of these lines between every other two; and through each point of bisection, draw a line parallel to $g\ l$ between the two adjacent bed-lines; and Fig. $A\ E\ F\ G$, as shown, will be the development of the intrados, answering to the middle development in which that of the stones are rectangles.

Draw $a\ e$ (Fig. 2) parallel to $A\ C$; with $a\ e$ as a chord, and with $I\ A$, the radius of the arc $A\ B\ C$, describe the arc $a\ c\ e$; draw $a\ f$ parallel to $A\ G$, and $e\ f$ parallel to $K\ F$; draw $f\ m$ parallel to $l\ g$, and draw $e\ m$ parallel to $A\ G$; bisect $a\ e$, $e\ f$, $f\ m$, each at the point 2; draw 2 c perpendicular to $a\ e$, 2 h perpendicular to $e\ f$, and 2 k perpendicular to $f\ m$; make 2 h, 2 k, each equal to 2 c; describe the arcs $e\ h\ f$, $f\ k\ m$; then No. 1, No. 2, No. 3, are the templets for working the stones, as before shown.

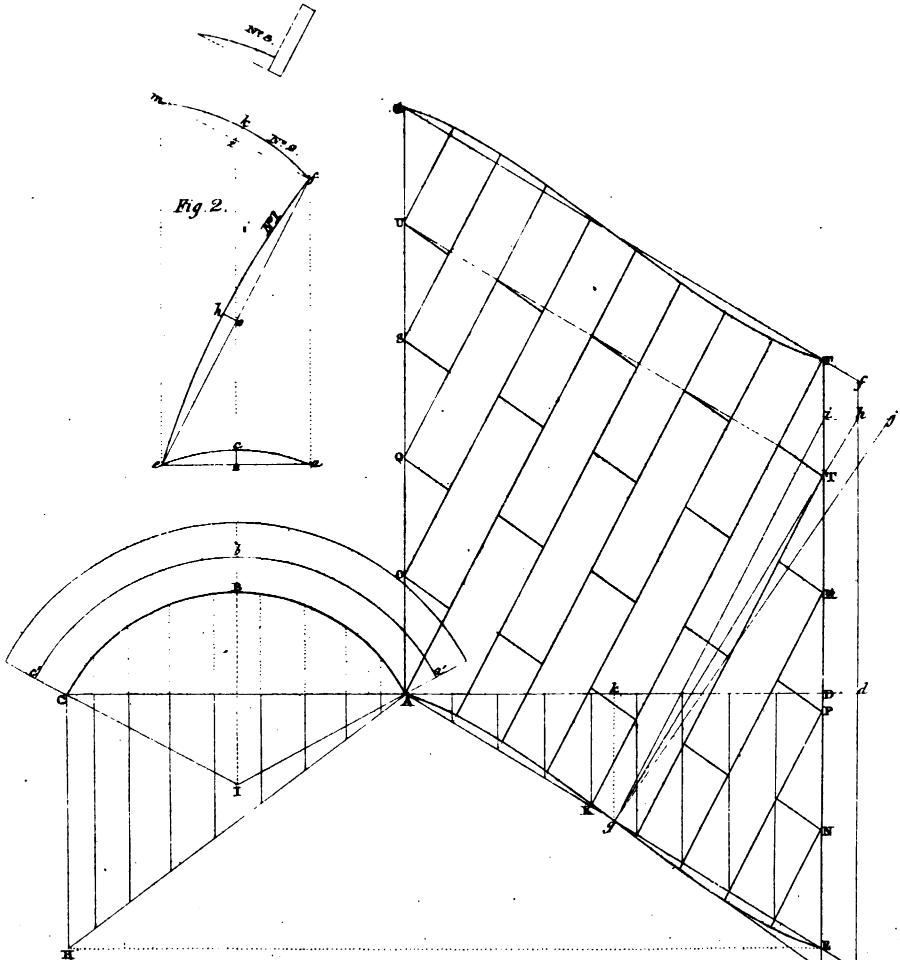


Fig. 2.
Nº 3.
Nº 2.
Nº 1.
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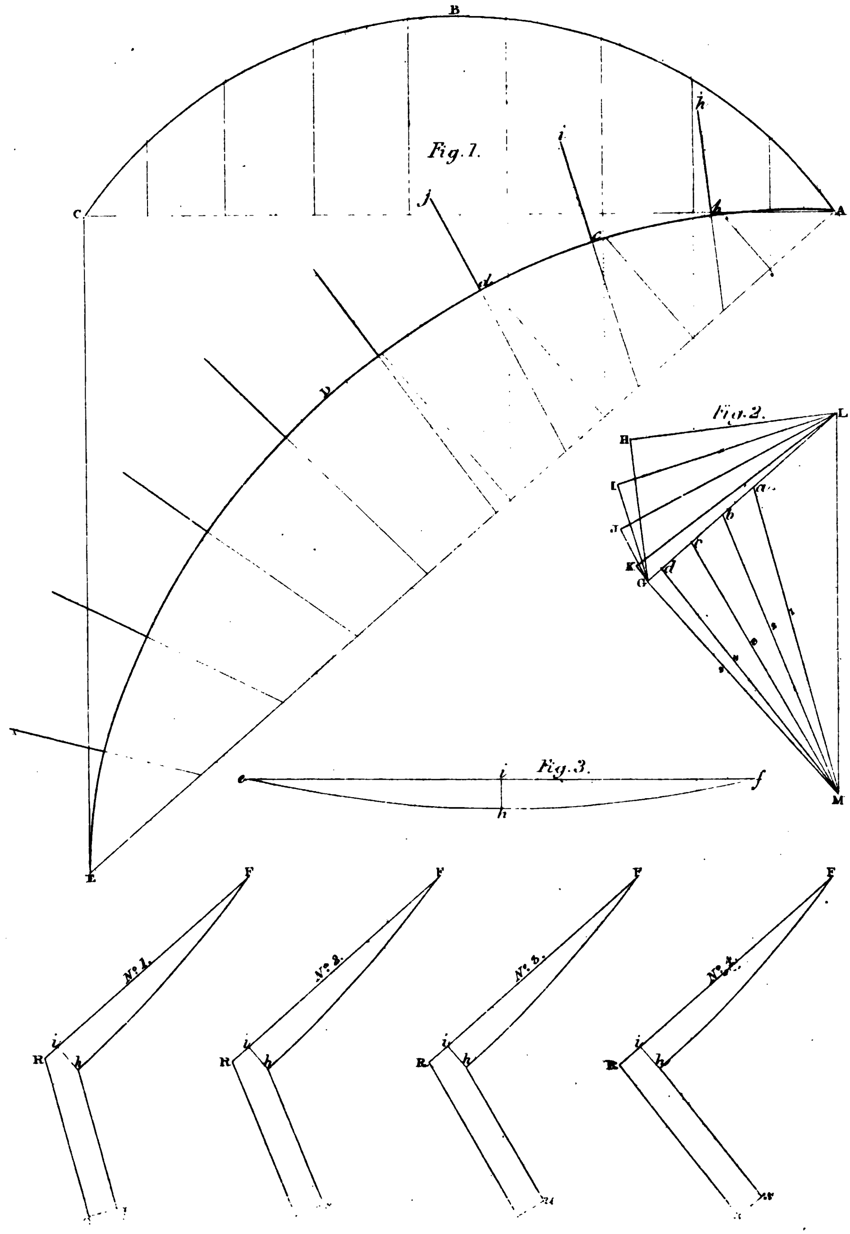
Fig. 1.
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j
Fig. 2.
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I
J
K
G
L
M
a
b
c
d
Fig. 3.
e
i
f
h
F
N° 1.
N° 2.
N° 3.
N° 4.
R
i
h

PROB. IX.—To find the curved bevels for cutting the quoin-heads of an oblique arch.

Let *A B C* (Fig. 1) be the right section; draw *C E* perpendicular to *A C*, and draw the straight line *A E*, making the angle *C A E* equal to the complement of the angle of obliquity; find the oblique section *A D E* of the cylinder (as in Prob. XIV, page xxi, Introduction); and the curve *A D E* is the common section of the concave cylindric surface, and the plane of the face of the arch. Divide the arc *A D E* into as many equal parts as the ring-stones are in number; and through the points of division draw *b h*, *c i*, *d j*, &c. perpendicular to the curve line *A D E*.

Parallel to the chord line *A E*, draw the straight line *G L*, (Fig. 2,) the points *G*, *L*, being taken at any convenient distance from each other; make the angle *G L M* (Fig. 2) equal to the acute angle *C E A*, (Fig. 1,) which the axis of the cylinder makes with the sectional line or chord; draw *G H*, *G I*, *G J*, &c. (Fig 2,) respectively parallel to *b h*, *c i*, *d j*, &c. (Fig. 1); draw *L H*, *L I*, *L J*, &c. (Fig. 2,) respectively perpendicular to *G H*, *G I*, *G J*, &c.; and the lines *L H*, *L I*, *L J*, &c. will be respectively parallel to tangents to the curve *A D E*, at the points *b*, *c*, *d*, &c. (Fig. 1.)

In *G L* make *G a*, *G b*, *G c*, &c. respectively equal to *G H*, *G I*, *G J*, &c.; draw *G M* perpendicular to *G L*; and join *a M*, *b M*, *c M*, &c. Then the angles *G a M*, *G b M*, *G c M*, &c. shall be the acute angles which tangent planes to the curved surface of the cylinder make with the plane of the oblique face *A D E*, at the points *b*, *c*, *d*, &c. (See Prob. XXXVI, page lxvii, Introduction.)

Let Fig. 3 be the curve of the bed-lines (found as in No. 1, Prob. V, page 8.) Then in each of the bevels, No. 1, No. 2, No. 3, &c. the part *F h i* is identical to the half *f h i*, (Fig. 3,) the straight side *R F* of each of the bevels being parallel to *G L*, (Fig. 2,) and the inner edges *h q*, *h s*, *h u*, &c. respectively parallel to *a M*, *b M*, *c M*, &c. (Fig. 2). The curved edge *h F* applies upon the bed-lines which are concave, and either of the straight edges of the parallel side serves for drawing a line across the bed, which, when the stone is set, shall be in the plane of the face of the arch.

The bevels No. 1, No. 2, No. 3, &c. are those which apply upon the bed-lines, where the soffit or intrados forms obtuse angles with the face of the arch; and the supplements of these bevels or angles apply upon the bed-lines, where the intrados forms acute angles with the said face.

Let us now suppose that the arch-stones are all set, except those which are required to form the quoin-heads, and that the stones which are to form the quoin-heads have been wrought, except the heads or ends which are to form the face of the arch. Then, in order to find how much is to be cut off each stone in every course, in order to complete that course, measure the distance of the remaining part of each bed-line upon the development, from the end of the last stone in that course to the curve in which each bed-line terminates, and apply the distances thus taken upon the arris-lines of the soffit from the end which forms the joint; and the length thus marked off upon each bed-line will give the extremities of these lines in the face of the arch. Then by the two bevels adapted to forming the head of that course, and with the proper bevel for each bed-line, draw a line with the parallel side across the bed, the curved edge of the other side being upon the arris of the stone which forms the bed-line. The superfluous part of the stone being properly cut away between the two lines thus drawn, shall, when the stone is set, be in the plane of the face of the arch.

It must here be observed, that, by the principle of the trihedral, the bed and joint-lines on the face are perpendicular to the curve, which is the intersection of the cylindric surface, and the plane of the face; but according to the principles of spiral surfaces, the joints on the face are curves, deviating slightly from straight lines, and which do not divide the curve of intersection exactly into equal parts. However, if the principle be carefully attended to in the execution, very little correction will be found necessary, the method being a near approximation. Its simplicity is ample compensation for its introduction.

The faces of the arch-stones of the oblique arch at Gateshead were cut upon this principle, the angle of obliquity being 76° 42′. The work was conducted by Mr. John Battey. The bevels of the stones, after the first cutting, wanted no correction.

ANOTHER METHOD OF FORMING THE QUOIN-HEADS,

Derived from the principle of the spiral construction.

Fig. 1 exhibits the right section, as also the oblique section or elevation, together with the development of the intrados; Fig. 2 the development of the extrados; and Fig. 3 the templet of the curvature of the bed-lines. The oblique section is found as in Prob. XIV, page xxi, Introduction, and the two developments as in Prob. IV, page 7.

The lines in the oblique section, which represent the joints in the beds, are found in the following manner:—Let h s v r...I (Fig. 1) be the curve-line which terminates one end of the development of the intrados, corresponding to the sectional-line $H\ I$ or $J\ a'$ on the plan. Prolong $C\ A$ to meet h F in D; and through the points s v r, &c. which are the ends of the developments of the bed-lines, draw s d, v d, r d, &c. parallel to h F, intersecting the bed-lines respectively in u, q, l, &c. Let the development of the first or shortest bed-line s k meet h F in k. Then, because s k, v u, r q, &c. are parallel and equi-distant, the parallel-lines h k, s u, v q, &c. are equal to one another. From C (Fig. 1) upon $C\ H$, set off distances respectively equal to d s, d v, d r, &c.; from the points of section in $C\ H$, draw lines parallel to $C\ A$ to meet the sectional-line $H\ I$ of the oblique face; and from the points of meeting draw lines perpendicular to $H\ I$, meeting the lower curve $I\ M\ H$ in the points s, v, r, &c. In the development of the extrados, (Fig. 2,) the points b, c, e, &c. are the ends of the bed-lines which terminate in the curve. Perpendicular to $a\ d$, draw $b\ i$, $c\ i$, $e\ i$, &c. meeting $a\ d$ in i, i, i, &c. Upon $c'\ J$ (Fig. 1) from c', set off distances respectively equal to $b\ i$, $c\ i$, &c. (Fig. 2). From the points thus marked off, draw lines parallel to $c'\ a$, or $C\ A$, to meet the sectional-line $J\ a'$; and from the points of meeting draw lines perpendicular to $J\ a'$, meeting the curve of the extrados in the points S', V', R', &c. Join S' s, V' v, R' r, &c. which are the representation of the intersections of the beds in the oblique section of the arch.*

Let Fig. 4 be the representation of a portion of the end of the cylinder adjacent to one of the oblique faces, which is represented by

* The reader must observe, that the lines S s, V v, R r, &c. which represent the joint-lines between the ring-stones, are not absolutely straight lines, but curves.

the curve *H S V R...I*, and which, by hypothesis, is identical to the semi-ellipse *H* s v r...*I* (Fig. 1). Let *S K, V U, R Q*, &c. (Fig 4,) represent portions of the first, second, third, &c. of the spiral bed-lines; and let *H K, S U, V Q*, &c. be supposed to be drawn on the surface of the cylinder parallel to the axis; and let *H Y, S T, V P*, &c. be parallel to *H I*. Then the straight lines *H K, S U, V Q*, &c. are by hypothesis equal to h k, s u, v q, &c. (Fig. 1,) and are consequently equal to one another; moreover the angles *K H Y, U S T, Q V P*, &c. are each equal to the angle of obliquity *C H I* (Fig. 1).

Prolong *S* s (Fig. 1) to y, *V* v to t, *R* r to p, &c. and draw s t, v p, r o, &c. parallel to *H I*. Let *S* s meet *H I* in y; and let *H Y, S T, V P*, &c. (Fig. 4,) be supposed to be made respectively equal to *H* y, s t, v p, &c. (Fig. 1); and let *S Y, V T, R P*, &c. (Fig. 4,) be joined; then by hypothesis *S Y, V T, R P*, &c. are respectively equal to s y, v t, r p, &c. (Fig. 1). Without repetition, it may be recollected that in each of the triangles represented by *K H Y, U S T, O V P*, &c. (Fig. 4,) two sides and the contained angle are given; therefore in each respective triangle the third side may be found, and in each of the triangles represented by *K Y S, U T V, Q P R*, &c. the three sides are given; hence the angles represented by *K S Y, U V T, Q R P*,* &c. may be found.

It may be readily admitted, that a short length of the development of a spiral-line will not differ sensibly from the length of the line itself; therefore the lines s k, v u, r q, &c. (Fig. 1,) which are portions of the developments of the bed-lines, will, in practice, be equal to equal lengths of the spirals themselves.

Draw *h k* (No. 1) in any convenient situation; make the angle *k h y* equal to the angle *C H I* (Fig. 1); and make *h k* (No. 1) equal to h k (Fig. 1). Make *h y* (No. 1) equal to *H* y, (Fig. 1,) and join *k y* (No. 1). With the distance y s (Fig. 1) from *y*, (No. 1,) describe an arc at *s*; and with the distance k s (Fig. 1) from *k*, (No. 1,) describe another arc intersecting the former in *s*. Join *y s*, and prolong *y s* to *a*.

Draw *s u* (No. 2) in any convenient situation; make the angle *u s t* equal to the angle *C H I*, (Fig. 1,) and make *s u* (No. 2) equal to s u (Fig. 1). Make *s t* (No. 2) equal to s t, (Fig. 1,) and join *u t* (No. 2).

* The angles represented by *K S Y, U V T, Q R P*, &c. are respectively contained by the curve-lines *K S, U V, Q R*, &c. and the straight lines *S Y, V T, R P*, &c. and therefore represent the complements of the angles made by the bed-lines on the soffit, and on the face or elevation of the arch.

With the distance t v (Fig. 1) from *t*, (No. 2,) describe an arc at *v*; and with the distance u v (Fig. 1) from *u*, (No. 2,) describe another arc intersecting the former in *v*. Join *t v*, and prolong *t v* to β.

Draw *v q* (No. 3) in any convenient situation. Make the angle *q v p* equal to the angle *C H I*, (Fig. 1,) and make *v q* (No. 3) equal to v q (Fig. 1). Make *v p* (No. 3) equal to v p, (Fig. 1,) and join *q p* (No. 3). With the distance p r (Fig. 1) from *p*, (No. 3,) describe an arc at r; and with the distance q r (Fig. 1) from *q*, (No. 3,) describe another arc intersecting the former in *r*. Join *p r*, and prolong *p r* to γ.*

With the mould of curvature of the bed-lines, draw the curve-line *k s*, (No. 1,) the curve-line *u v*, (No. 2,) the curve-line *q r*, (No. 3,) &c.; then the angle *k s a*, (No. 1,) contained by the curve-line *k s*, and the straight line *s a*, is the bevel adapted to the first joint; the angle *u v* β, (No. 2,) contained by the curve-line *u v* and the straight line *v* β, is the bevel adapted to the second joint; the angle *q v* γ, (No. 3,) contained by the curve-line *q r* and the straight line *r* γ, is the bevel adapted to the third joint, and so on.

In each case the curved edge must be applied along the arris of the stone between the soffit and bed, and the straight edge across the bed, in order to draw the line which, when the stone is cut, the surface shall be in the plane of the face.

* The reader will no doubt have observed, that the lines *h k*, (No. 1,) *s u*, (No. 2,) *v q*, (No. 3,) must be equal to each other, because the lines h k, s u, v q, &c. (development of the intrados,) are equal to each other; and that the lines *h y*, (No. 1,) *s t*, (No. 2,) *v p*, (No. 3,) &c. are respectively equal to h y, s t, v p, &c. (oblique section); moreover, that the *y s*, (No. 1,) *t v*, (No. 2,) *p r*, (No. 3,) &c. are respectively equal to y s, t v, p r, &c. (oblique section); and that the lines *k s*, (No. 1,) *u v*, (No. 2,) *q r*, (No. 3,) &c. are respectively equal to k s, u v, q r, &c. (development of the intrados.)

ANOTHER METHOD

Practised by workmen, is to set up two staves or straight edges perpendicular to the horizon, in the plane of the face or elevation of the arch. Draw all the bed-lines upon the boarding. Prepare two moulds, as shown at No. 3, No. 4, Plate XXV; the one, No. 3, representing a section of the stone,* and the other, No. 4, the bed.† Make moulds to the developments of the ends of the boards, and these moulds will serve for drawing the ends of the stones upon the soffit. In order to find the moulds for cutting any particular stone, so as to range in the plane of the elevation of the arch, place the section No. 3 as nearly at right angles to one of the bed-lines, and as nearly to the plane of the elevation as possible; then place the mould No. 4, so that the under edge *n o p* may be upon the bed-line, and the side resting upon the edge of No. 3, the two planes being perpendicular to each other. Place a straight edge upon the face of No. 4, so that the edge may be upon the end of the bed-line which is in the plane of elevation, and the straight edge out of winding with the straight edge of one of the staves. In this position draw a line along the straight edge upon the face of the board; and the line thus drawn will make an angle with the bed-line, which is a curve nearly equal to the angle which the line on the end of the bed of the stone makes with the bed-line.

In this manner the face of every stone, so as to range in the plan of elevation, may be formed.

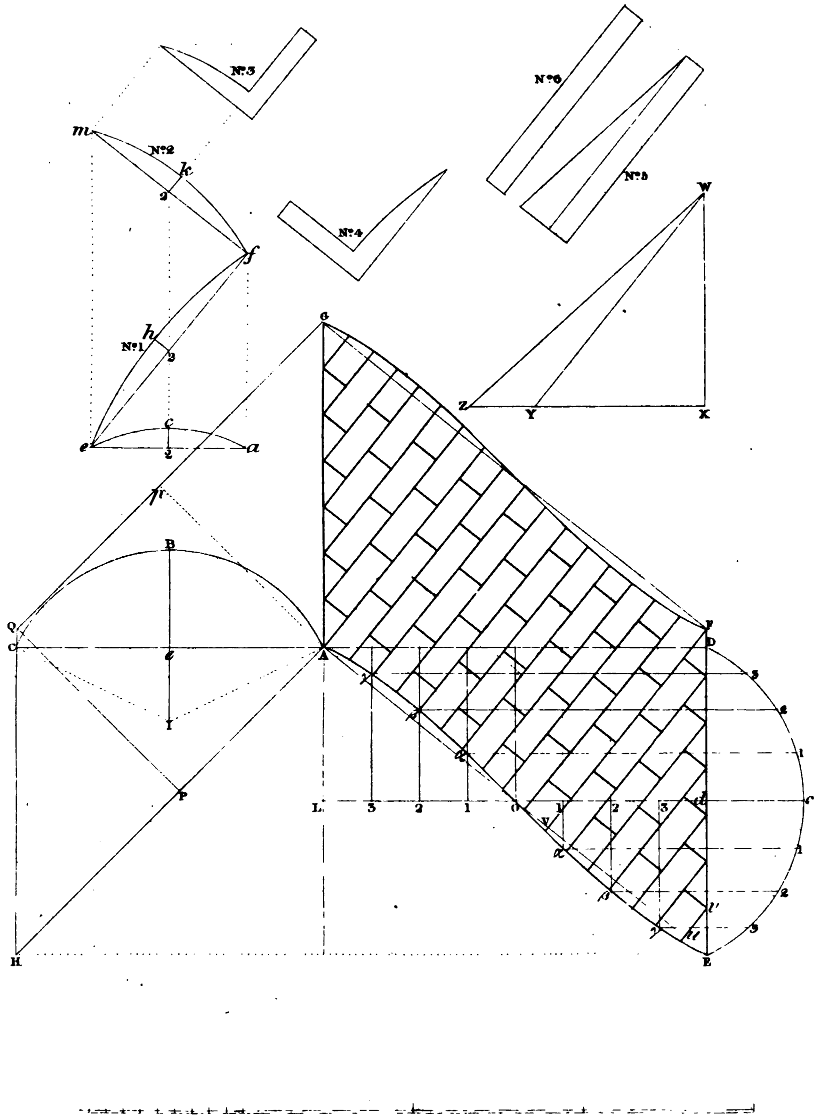
PROPOSITION.—To construct an oblique arch, the angle of obliquity being 45°, the span of the oblique face or length of the front $25\frac{1}{2}$ feet, the height of the intrados $5\frac{3}{4}$ feet, the distance between the faces $13\frac{1}{2}$ feet, and the number of spiral courses 17.

Draw the straight line $A\ H$, and make $A\ H$ equal to 25·5 feet. Make the angle $A\ H\ C$ equal to the angle of obliquity, that is, equal to 45°. Draw $A\ C$ perpendicular to $H\ C$, bisect $A\ C$ in e, and

* The mould No. 3 is not absolutely the section of one of the stones, as made by a plane perpendicular to one of the bed-lines, but nearly so.

† Nor is the mould No. 4 the bed of the stone, which is a spiral surface, and therefore not a plane. The practice by this method is therefore a near approximation, as in the other two preceding methods.

PLATE 29



draw $e\ B$ perpendicular to $A\ C$. Make $e\ B$ equal to 5·875 feet, and through the three points A, B, C, describe the arc of a circle. Prolong $C\ A$ to D, and make $A\ D$ equal to the length of the arc $A\ B\ C$. Through D draw $E\ F$ perpendicular to $A\ D$, and make $D\ E$ equal to $C\ H$. Upon $D\ E$ describe the segment $D\ c\ E$ of a circle similar to the segment $A\ B\ C$ of the right section, which in this case, because $A\ C$ and $D\ E$ are equal, the segment $D\ c\ E$ is equal to the segment $A\ B\ C$.

Bisect the arc $D\ c\ E$ in c, and from c draw the straight line $c\ L$ parallel to $D\ A$, intersecting $D\ E$ in d. Make $d\ L$ equal to $D\ A$, and bisect $d\ L$ in o. Divide each of the arcs $c\ E$, $c\ D$, and each of the straight lines $o\ d$, $o\ L$, into any number of equal parts, (as here into four,) at the points 1, 2, 3, &c. From the points 1, 2, 3, &c. in each of the arcs parallel to $d\ L$, draw 1 α, 2 β, 3 γ, &c.; and from the points 1, 2, 3, &c. in each of the straight lines parallel to $D\ E$, draw 1 α, 2 β, 3 γ, &c. From A, through the points of intersection to E, draw the curve $A\ \gamma\ \beta\ \alpha\ 0\ \alpha\ \beta\ \gamma\ E$, which is the development of the intersection of the plane of the face of the arch, and the cylindric surface, or the projection of a spiral, of which the length is equal to the length of the arc of the right section, and the breadth equal to the distance of obliquity.

Draw $A\ G$ perpendicular to $A\ C$, and prolong $H\ C$ to Q. Draw $A\ p$ perpendicular to $A\ H$, and make $A\ p$=13·5 feet, the distance between the faces of the arch. Through p draw $G\ Q$ parallel to $A\ H$, and draw $G\ F$ parallel to $A\ E$. Then the parallelogram $A\ H\ Q\ G$ is the plan of the arch. Make the curve $G\ F$ the same as the curve $A\ E$; and the curve lines $A\ E$, $G\ F$, will be the development of the curves formed by the intersections of the faces of the arch, and the development of the surface.

Draw $F\ V$ perpendicular to $A\ E$, meeting the straight line $A\ E$ in V; and if the straight line $A\ E$ be divided into seventeen equal parts, it will be found that $A\ V$ will contain ten nearly, and $V\ E$ seven nearly. Therefore, divide each abutment, $A\ G$ and $E\ F$, into seven equal parts. Draw lines from each of the points in each abutment parallel to $F\ V$, to meet the curve-line $F\ G$ at one end, and the curve $A\ E$ at the other. Also, divide the distance $A\ V$ into ten equal parts, and through each point of division parallel to $V\ F$, draw a line to meet each of the opposite curves, and the whole of the bed-lines will thus be drawn. From each of the seven points draw lines as in the figure parallel to $A\ E$, leaving the alternate portions blank; the remaining ones will represent the ends of the courses.

TO MAKE THE TWISTING RULES.

Draw *X W* parallel to *A G*, and *X Y* parallel to *D A*; make *X Y* equal to the radius *I A* or *I C*, and draw *Y W* parallel to *V F*; prolong *X Y* to *Z*, and make *Y Z* equal to four feet, the breadth of the beds; join *Z W*; and the angle *Y W Z* is the angle of the twist.

The length of each of the twisting rules must be equal to the length of the stones. The figure of one of them is a trapezoid, and the two straight edges in the length are inclined to each other, at an angle which is equal to the angle of the twist; or if a line be drawn on one of the faces parallel to one of the edges at the breadth of the narrower end, the line thus drawn shall cut off a parallelogram or rectangle, and will make an angle with the remaining edge equal to the angle *Y W Z*, the angle of the twist. The other rule is a rectangle of the same breadth as the narrow end of the tapering rule, which is generally about three inches.

TO CONSTRUCT THE TEMPLETS FOR WORKING THE STONES.

Draw *a e* parallel to *A C* of any convenient length, and with the radius *I A* or *I C* describe the arc *a c e*. Draw *a f* and *e m* parallel to *A G*. Draw *e f* parallel to *V F*, and *f m* parallel to *E A*. Bisect the straight line *a e* in 2, and draw 2 *c* perpendicular to *a e*, meeting the arc *a c e* in *c*. Bisect *e f* in 2, draw 2 *h* perpendicular to *e f*, make 2 *h* equal to 2 *c*, and through the three points *e*, *h*, *f*, describe the arc of a circle, which will be the curve of the bed-lines. Bisect the straight line *f m* in 2, draw 2 *k* perpendicular to *f m*, make 2 *k* equal to 2 *c*, and through the three points *f*, *k*, *m*, describe the arc of a circle. Make two templets, No. 1, No. 2, to form the segments *f h e*, *f k m*. Make also two arch-squares, No. 3, No. 4, having the inner edge of one limb made to the circular arc, and the other a straight tending to the centre of the circle, of which the curve is an arc. The arch-square No. 3 is for working the soffit, the bed being previously wrought; or for working the bed, the soffit being previously wrought. The arch-square No. 4 is to find the direction of the joints. The curved edge of the limb No. 3 is the same as the half-segment No. 2, and the curved edge of the limb No. 4 is the same as the half-segment No. 1.

The method of cutting the quoin-heads is explained in Prob. IX, page 19. See also page xxxvii, Introduction, where the principle is explained.

REMARK.

The oblique arch required to be constructed by this proposition is the model, No. 38, exhibited in the Polytechnic Exhibition, which was held in Newcastle-upon-Tyne in 1840. It was constructed in plaster by three of my pupils belonging to the School of Fine Arts, namely, Thomas Bryson and Lancelot Armstrong, masons, and William Tate, joiner, for the purpose of competition, and shewing the principle of working the arch-stones. The templets are made of wood, which are placed upon the model. The scale is an inch to the foot.

The above model has only sixteen spiral courses, although seventeen were intended. However, the calculations in all the principal parts will be the same.

The following gratifying letter on this subject was forwarded to me from the Rev. E. D. Rendell, Secretary to the Exhibition:—

"Academy of Arts, *June* 13*th*, 1840.

"Sir—I have pleasure in informing you, that the committee for awarding the premiums for models now exhibiting in the Newcastle Exhibition, have awarded to your three pupils, namely, Thomas Bryson, mason, William Tate, joiner, and Lancelot Armstrong, mason, for the construction of the model of an oblique arch, with the motto, "Whatever science discovers to be useful, let it be known," the premium of five pounds; and also in stating that I shall have pleasure in paying that sum to them on Saturday, the 20th instant, if they will call on me for that purpose, at the same time producing this letter.—I am, Sir, yours respectfully,

"E. D. RENDELL, Secretary."

In this plate, and in each of the three following plates, the parallelogram $A\ H\ Q\ G$ is the plan of the arch-way, $A\ G$, $H\ Q$, being the length of the abutments, and $A\ H$, $G\ Q$, the lengths of the two faces or elevations. Draw $Q\ P$ perpendicular to $A\ H$, meeting $A\ H$ in P, and $P\ Q$ is the distance between the front and rear faces.

The figure $A\ E\ F\ G$ is the development of the intrados, $A\ G$, $E\ F$, the springing-lines, and $A\ D$ perpendicular to $E\ F$, contained between the parallels $A\ G$, $E\ F$, is equal to the length of the arc $A\ B\ C$. The straight line $A\ E$ is the length of each spiral-line in which the joints are arranged. Draw $F\ V$ perpendicular to $A\ E$, meeting $A\ E$ in V; $E\ V$ is the breadth of the springing courses; moreover $A\ V$ is the breadth of the entire courses. It is evident in the development, that since the bed-lines are all perpendicular, and the joint-lines parallel to $A\ E$, the springers are equal right-angled triangles, each similar to the triangle $E\ D\ A$, viz. the triangle $E\ u\ v$ to the triangle $E\ D\ A$.

The construction of the development of the intrados has already been explained.

In the triangle $A\ C\ H$, right-angled at C, are given the angle $A\ H\ C=45°$, the hypothenuse $A\ H=25\frac{1}{2}$ feet, to find $A\ C$ and $H\ C$.

rad. 1 : sin. 45 :: $A\ H : A\ C$.

$A\ C=\text{sin. } 45° \times A\ H=\cdot70711 \times 25\cdot5=18\cdot0313=H\ C=D\ E=$ 18 feet nearly, the distance of obliquity.

In the right-angled triangle $H\ P\ Q$, are given the angle $P\ H\ Q$ or $A\ H\ Q=45°$, and the opposite side $P\ Q=13\cdot5$, to find $H\ Q=A\ G=E\ F$.

sin. 45° : rad.=1 :: $P\ Q : H\ Q$, the length of the springing-line, or $\cdot70711 : 1 :: 13\cdot5 : H\ Q=\dfrac{13\cdot5}{\cdot70711}=19\cdot091=19\cdot1$ nearly, the length of the springing-line.

From the chord equal to 18·0313 feet, and the height of the arc= $5\frac{7}{8}$ feet=5·875 feet, we shall find the radius of the circle equal to 9·85 feet nearly, and the length of the arc 22·766 feet=$A\ D$.

$A\ D^2=518\cdot290$ nearly.

$D\ E^2=325\cdot1277$ nearly.

$\therefore A\ E^2=843\cdot4177$ nearly.

$\therefore A\ E=\sqrt{(843\cdot4177)}=29\cdot0$ feet nearly.

Let r be the radius of the cylinder, R the radius of curvature of the joint-line spirals, and $R_{\prime}$ the radius of curvature of the bed-line spirals.

Then $A\ E^2 \times r=843\cdot4177 \times 9\cdot85=8307\cdot664345$.

$$R=\frac{A\ E^2 \times r}{A\ D^2}=\frac{8307\cdot664345}{518\cdot29}=16\cdot02 \text{ feet nearly.}$$

$$R_{\prime}=\frac{A\ E^2 \times r}{D\ E^2}=\frac{8307\cdot664345}{325\cdot1277}=25\cdot51 \text{ feet nearly.}$$

By similar triangles, $A\ E\ D$, $F\ E\ V$:—

$$A\ E : E\ D :: F\ E : E\ V=\frac{E\ D \times F\ E}{A\ E}=\frac{18 \times 19}{29}=11\cdot78 \text{ feet nearly.}$$

Then as $E\ A$ is to $V\ E$, so is the entire number of spiral courses to the number of courses in each abutment.

Or 29 : 11·78 :: 17 : $A\ V=6\cdot9$, which is nearly equal to seven.

Therefore 11·78÷7=1·68 feet, the thickness of the springing courses.

11·78 feet subtracted from 29 feet, gives 17·22 feet, the breadth of the entire courses.

17·22÷10=1·722, the thickness of the entire course.

XIX

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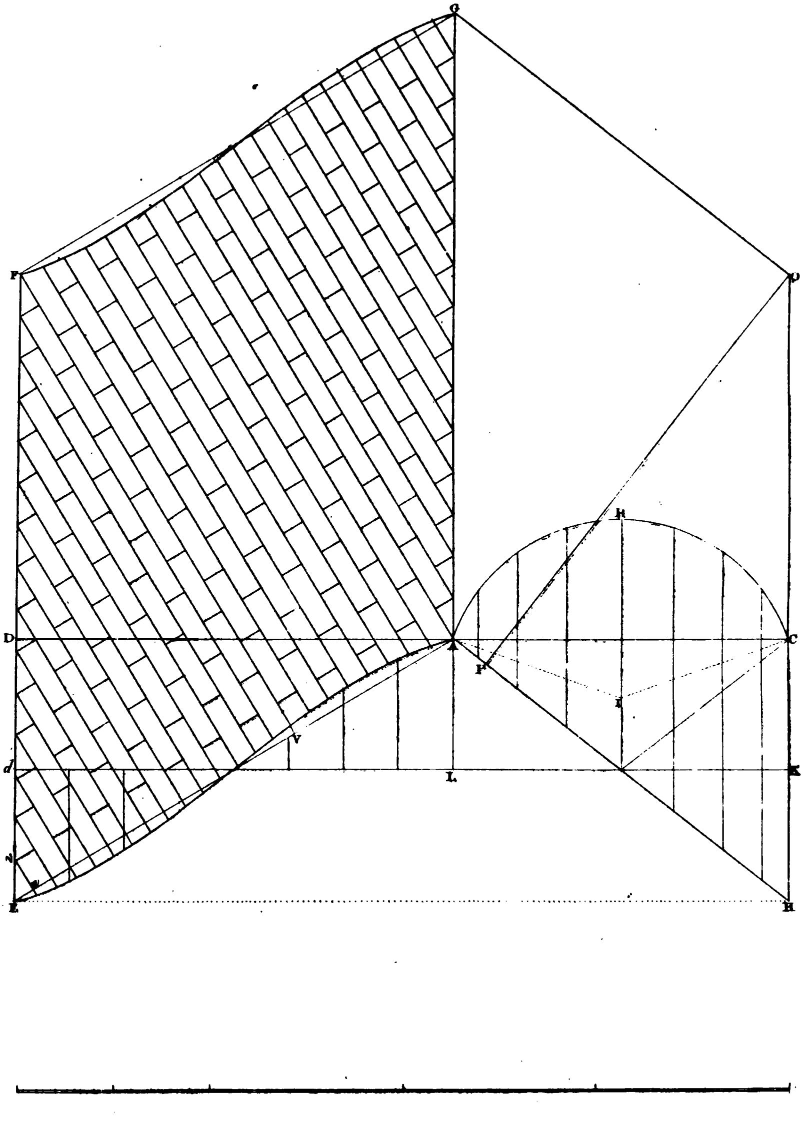
PLATE 30.



EXAMPLE I.—Oblique Arch over Willington Waggonway, upon the Newcastle and North Shields Railway.

DESIGNED BY ROBERT NICHOLSON, CIVIL ENGINEER.

In this example are given the distance of obliquity equal to 13·63 feet, the width of the right section of the arch-way equal to 17·374 feet, the height of the intrados equal to 6 feet, the length of each springing-line equal to $32\frac{1}{2}$ feet, and the number of courses equal to 23; to find the requisites for executing the work.

From the chord 17·374 feet, and the height 6 feet of the arc, will be found the radius 9·29 feet, (Introduction, page xliii, Prob. XXV,) and the length of the arc 22·46 feet (Introduction, page xli, Prob. XXIV.)

In the triangle $A\,D\,E$, right-angled at D, are given $A\,D = 22{\cdot}46$ feet, and $D\,E = 13{\cdot}63$ feet, to find $A\,E$, and thence the radii of curvature.

Now, $A\,D^2 = 22{\cdot}46^2 = 504{\cdot}4516 = 504{\cdot}45$ feet nearly.

And $D\,E^2 = 13{\cdot}63^2 = 185{\cdot}7769 = 185{\cdot}78$ feet nearly.

$\therefore A\,E^2 = A\,D^2 + D\,E^2 = 690{\cdot}2285 = 690{\cdot}23$ feet nearly.

$A\,E = \sqrt{(690{\cdot}2285)} = 26{\cdot}27$ feet nearly.

Let $r = 9{\cdot}29$ feet, the radius of the cylinder, R equal the radius of curvature of the joint-line spirals, R' equal to the radius of curvature of the bed-line spirals.

$$\text{Then } R = \frac{A\,E^2 \times r}{A\,D^2}, \text{ and } R' = \frac{A\,E^2 \times r}{D\,E^2}.^*$$

Now, $A\,E^2 \times r = 690{\cdot}23 \times 9{\cdot}29 = 6412{\cdot}24$ nearly;

Therefore $R = 6412{\cdot}24 \div 504{\cdot}45 = 12{\cdot}6$ feet nearly,

And $R' = 6412{\cdot}24 \div 185{\cdot}78 = 34{\cdot}5$ feet nearly.

By similar triangles, $A\,E\,D,\ F\,E\,V;$

$$A\,E : E\,D :: F\,E : V\,E = \frac{E\,D \times F\,E}{A\,E} = \frac{13{\cdot}63 \times 32{\cdot}5}{26{\cdot}27} = 16{\cdot}86 \text{ feet.}$$

$A\,V = A\,E - V\,E = 26{\cdot}27 - 16{\cdot}86 = 9{\cdot}41$ feet.

Then as $A\,E$ is to $V\,E$, so is the entire number of courses to the number of springers or springing courses.

Let $a = 23$ be the entire number of courses, and x the number of springers required, $A\,E : E\,V :: a : x = \dfrac{E\,V \times a}{A\,E} = \dfrac{16{\cdot}86 \times 23}{26{\cdot}27} = 14{\cdot}76.$

But as 14·76 is nearer to 15 than to 14, let the number of springers be 15; and the 15 springers will have 15 arch-stones for the heads of the courses which they support.

Now, $23 - 15 = 8$, the number of courses independent of the springing courses.

Or $15 + 8 = 23$, the entire number of courses.

$16{\cdot}86 \div 15 = 1{\cdot}124$ feet, for the thickness of the springing courses.

And $9{\cdot}41 \div 8 = 1{\cdot}176$ feet, the thickness of the independent courses.

* See Introduction, page lx, Prob. XXXIV.

EXAMPLE II.—Oblique Arch over High Street, Gateshead, upon the Brandling Junction Railway.

DESIGNED BY JOHN AND BENJAMIN GREEN, ARCHITECTS.

In this example are given the distance of obliquity equal to 9·166 feet, the width of the right section equal to 38·75 feet, the height of the intrados equal to 6 feet, the width above the arch between the outsides of the parapets equal to 26 feet, and the number of courses equal to 29.

By similar triangles, $A\,C\,H$, $Q\,P\,H$; $A\,C : C\,H :: Q\,P : P\,H$.

$$\therefore P\,H = \frac{C\,H \times Q\,P}{A\,C} = \frac{9{\cdot}166 \times 26}{38{\cdot}75} = 6{\cdot}15 \text{ feet.}$$

$\therefore H\,Q = \sqrt{(P\,Q^2 + P\,H^2)} = \sqrt{(26^2 + 6{\cdot}15^2)} = 26{\cdot}71$ feet, which is the length of each springing-line or abutment.

From the chord equal to 38·75 feet, and the height of the arc equal to 6 feet, are found the radius equal to 34·28 feet, and the length of the arc $A\,B\,C$ equal to 41·19 feet, which is the breadth of the development of the intrados.

In the triangle $A\,D\,E$, right-angled at D, are given $A\,D = 41{\cdot}19$ feet, and $D\,E = C\,H = 9{\cdot}166$ feet, to find $A\,E$.

Now, $A\,D^2 = 41{\cdot}19^2 = 1696{\cdot}6161 = 1696{\cdot}62$ nearly.

And $D\,E^2 = 9{\cdot}166^2 = 84{\cdot}0155 = 84{\cdot}$ nearly.

$\therefore A\,E^2 = A\,D^2 + D\,E^2 = 1780{\cdot}6316 = 1780{\cdot}63$ nearly.

$\therefore A\,E = \sqrt{(1780{\cdot}6316)} = 42{\cdot}197 = 42{\cdot}2$ feet nearly, which is the length of the spiral joint-line on the intrados.

Now, $A\,E \times r = 1780{\cdot}63 \times 34{\cdot}28 = 61039{\cdot}9964 = 61040{\cdot}$ nearly.

$\therefore R = 61040 \div 1696{\cdot}62 = 35{\cdot}9 = 36$ feet nearly; and

$R' = 61040 \div 84 = 726{\cdot}7 = 726$ feet nearly.

By similar triangles, $A\,E\,D$, $F\,E\,V$; $A\,E : E\,D :: F\,E : E\,V$.

$$\therefore E\,V = \frac{E\,D \times F\,E}{A\,E} = \frac{E\,D \times A\,G}{A\,E} = \frac{9{\cdot}166 \times 26{\cdot}71}{42{\cdot}2} = 5{\cdot}8 \text{ feet nearly.}$$

Then $A\,E : E\,V :: a : x$.

$$\therefore x = \frac{E\,V \times a}{A\,E} = \frac{5{\cdot}8 \times 29}{42{\cdot}2} = 4 \text{ nearly, for the number of springers.}$$

Now, $29 - 4 = 25$, the number of whole courses; and $A\,V = A\,E - E\,V = 42{\cdot}2 - 5{\cdot}8 = 36{\cdot}4$ feet, which is the breadth of the development of the intrados of the entire courses.

$\therefore 5{\cdot}8 \div 4 = 1{\cdot}45$, the thickness of the springing courses; and

$36{\cdot}4 \div 25 = 1{\cdot}456$, the thickness of the entire courses.

By similar triangles, $E\,D\,A$, $E\,u\,v$; $E\,D : D\,A :: E\,u : u\,v$.

$$\therefore u\,v = \frac{D\,A \times E\,u}{E\,D} = \frac{41{\cdot}19 \times 1{\cdot}45}{9{\cdot}166} = 6{\cdot}51 \text{ feet nearly,}$$ which is the length of the inclined back of each springer; and $26{\cdot}71 \div 4 = 6{\cdot}677$, the length of the bed.

PLATE 31

Fig. 1

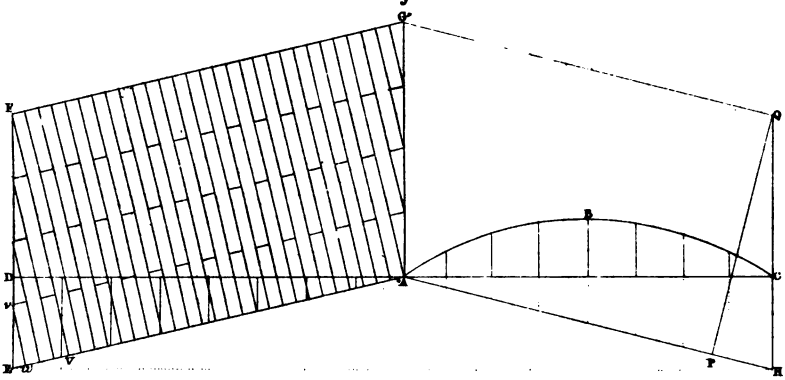


Fig. 2

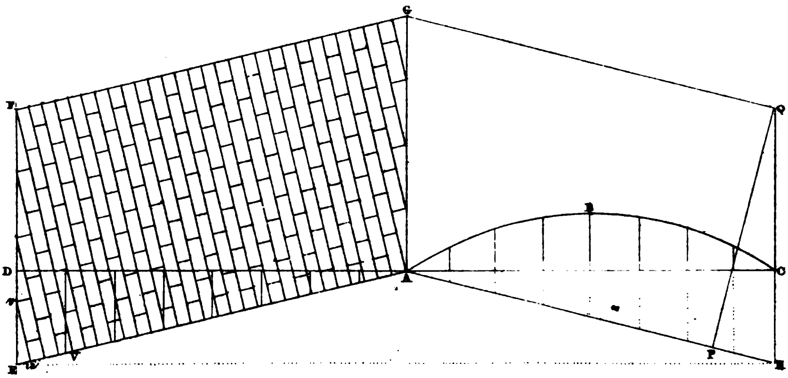
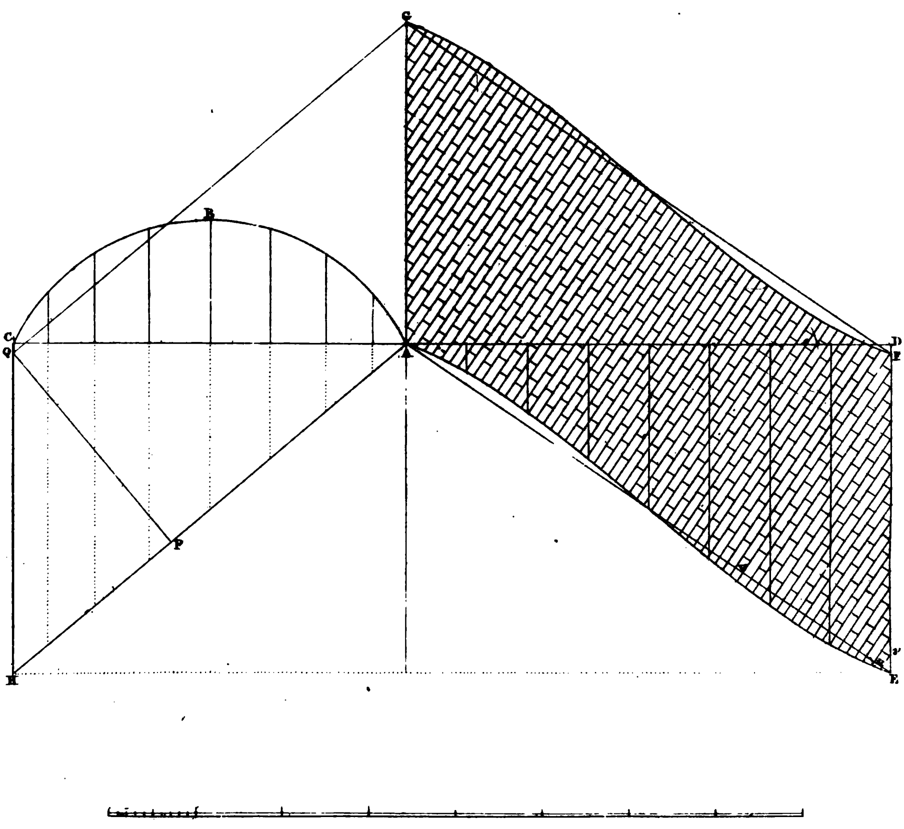


PLATE 32



EXAMPLE III.—Being one of the four Oblique Arches of the Bridge over the River Tees, upon the great North of England Railway, near Croft.

DESIGNED BY HENRY WELCH, CIVIL ENGINEER.

In this example are given the angle of obliquity equal to 50°, the span of the oblique face equal to 60 feet, the height of the intrados equal to 14·5 feet, the distance between the parapets 28·75 feet, and the breadth of the courses on the intrados equal to 13 inches.

In the triangle $A\,C\,H$, right-angled at C, are given the angle $A\,H\,C$ equal to 50°, and the hypothenuse $A\,H$ equal to 60 feet, to find $A\,C$ and $H\,C$.

rad. : sin. H° :: $A\,H : A\,C$.

$\therefore A\,C = \frac{\text{sin. } H^\circ \times A\,H}{\text{rad.*}} = \text{sin. } 50^\circ \times 60 = \cdot 7660 \times 60 = 45 \cdot 96$ feet nearly, and rad. : cos. H° :: $A\,H : H\,C$.

$\therefore H\,C = \frac{\text{cos. } H^\circ \times A\,H}{\text{rad.}} = \text{cos. } 50^\circ \times 60 = \cdot 6428 \times 60 = 38 \cdot 568$ feet nearly, which is the distance of obliquity.

In the right-angled triangle $H\,P\,Q$, are given in the angle $P\,H\,Q$ or $A\,H\,Q$ equal to 50°, and the opposite side $P\,Q$ equal to 28·75 feet, to find $H\,Q = A\,G = E\,F$.

sin. H° : rad. :: $P\,Q : H\,Q$.

$\therefore H\,Q = \frac{\text{rad.} \times P\,Q}{\text{sin. } H^\circ} = \frac{28 \cdot 75}{\text{sin. } 50} = \frac{28 \cdot 75}{\cdot 766} = 37 \cdot 53$ feet, which is the length of the springing-line; hence $A\,G = E\,F = 37 \cdot 53$ feet.

From the chord of the arc equal to 45·96 feet, and the height of the intrados equal to 14·5 feet, we shall find the radius of the circle equal to 25·46 feet nearly, and the length of the arc equal to 57·32 feet nearly.

$$\text{Now, } A\,D^2 = 57 \cdot 32^2 = 3285 \cdot 5824 = 3285 \cdot 58 \text{ nearly;}$$
$$\text{And } D\,E^2 = 38 \cdot 57^2 = 1487 \cdot 6449 = 1487 \cdot 64 \text{ nearly.}$$
$$\therefore A\,E^2 = A\,D^2 + D\,E^2 = 4773 \cdot 2273 = 4773 \cdot 23 \text{ nearly.}$$
$$\therefore A\,E = \sqrt{(4773 \cdot 2273)} = 69 \cdot 08 \text{ nearly.}$$
$$\therefore A\,E^2 \times r = 4773 \cdot 23 \times 25 \cdot 46 = 121526 \cdot 4358.$$
$$\therefore R = 121526 \cdot 44 \div 3285 \cdot 58 = 36 \cdot 99 \text{ feet nearly.}$$
$$\therefore R' = 121526 \cdot 44 \div 1487 \cdot 64 = 81 \cdot 69 \text{ feet nearly.}$$

By similar triangles, $A\,E\,D$, $F\,E\,V$;

$$A\,E : E\,D :: F\,E : E\,V = \frac{E\,D \times F\,E}{A\,E} = \frac{38 \cdot 568 \times 37 \cdot 54}{69 \cdot 08} = 20 \cdot 95 \text{ feet.}$$

* Radius=1.

Now, $69{\cdot}08 \times 12 \div 13 = 63{\cdot}7$ for the number of courses, say 63.

$69{\cdot}08 : 20{\cdot}95 :: 63 : x = \frac{20{\cdot}95 \times 63}{69{\cdot}08} = 19{\cdot}1 = 19$ nearly, which is the number of springers in each abutment.

$\therefore 37{\cdot}53 \div 19 = 1{\cdot}97$ feet nearly, for the length of the bed of each springer.

$E\,u = 20{\cdot}95 \times 12 \div 19 = 13{\cdot}23$ inches, which is the thickness of each of the springing courses, or the breadth of each springing course on the soffit.

Now, $63 - 19 = 44$, which is the number of entire courses; and $69{\cdot}08 - 20{\cdot}95 = 48{\cdot}13$ feet, is the breadth of the soffit of the entire courses.

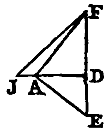
$\therefore 48{\cdot}13 \times 12 \div 44 = 13{\cdot}12$ inches, which is the thickness of the entire courses.

By similar triangles, $E\,D\,A$, $E\,u\,v$;

$E\,D : D\,A :: E\,u \therefore u\,v.$

$\therefore u\,v = \frac{D\,A \times E\,u}{E\,D} = \frac{57{\cdot}32 \times 13{\cdot}23}{38{\cdot}57} = 19{\cdot}66$ inches, which is the length of the inclined back.

TO FIND THE ANGLE OF THE TWIST.



In the triangle $A\,D\,E$, right-angled at D, are given $D\,E = 38{\cdot}57$, and $A\,D = 57{\cdot}32$, to find the angle $E\,A\,D$.

$A\,D : D\,E :: 1 : \tan.\ D\,A\,E = \frac{D\,E}{A\,D} = \frac{38{\cdot}57}{57{\cdot}32} = {\cdot}6728$ $= \tan.\ 33^\circ\ 56'$.

In the triangle $A\,D\,F$, right-angled at D, are given the side $A\,D = 25{\cdot}46 =$ the radius of the cylinder, and the angle $A\,F\,D$ equal to the angle $D\,A\,E = 36^\circ\ 56'$, to find $D\,F$.

$\tan.\ A\,F\,D : 1 :: A\,D : D\,F = \frac{A\,D}{\tan.\ A\,F\,D} = \frac{25{\cdot}46}{\tan.\ 36{\cdot}56} = \frac{25{\cdot}46}{6728} =$ $37{\cdot}84$ feet.

Suppose now the thickness of the arch to be 2 feet 6 inches $= 2{\cdot}5$; then $25{\cdot}46 + 2{\cdot}5 = 27{\cdot}96$, the radius of the exterior cylinder.

Produce the side $D\,A$ of the triangle $A\,D\,F$ to J; make $D\,J = 27{\cdot}96$, and join $A\,F$; then in the triangle $J\,D\,F$, right-angled at D, are given the side $J\,D = 27{\cdot}96$, and the side $D\,F = 37{\cdot}84$, to find the angle $J\,F\,D$.

$F D : D J :: 1 : \text{tan. } J F D = \frac{D J}{F D} = \frac{27{\cdot}96}{37{\cdot}84} = {\cdot}73{\cdot}89 = \text{tan. } 36° \; 37'.$

Hence $\angle J F D - \angle A F D = \angle A F J$, the angle of the twist;

Or $36° \; {\cdot}27 - 33° \; 56 = 2{\cdot}31'$, the angle of the twist.

But as measures in degrees, &c. are not adopted for the use of workmen, it will be more convenient to reduce the breadth, at a certain distance from the angular point, to inches.

Suppose the length of each of the winding rules to be 5 feet, and the breadth of the parallel one to be three inches, it will be—

$1 : \text{tan. } 2° \; 31 :: 5 :$ the additional breadth at the greater end; or

$1 : {\cdot}04395 :: 5 : {\cdot}21975$ feet$= 2{\cdot}64$ inches nearly.

$\therefore 3 + 2{\cdot}64 = 5{\cdot}64$ inches, which is the breadth of the greater end, 3 inches being that of the less end.

For the application of those straight edges, see page 9, as also pages 16 and 17. All the six templets are exhibited in Plate XXV, as No. 1, No. 2, No. 3, &c.

EXAMPLE FOR PRACTICE.

In this example are given the length of the oblique face, equal to 42 feet, the width of the right section of the arch-way equal to 19 feet, the height of the intrados equal to 7 feet 1 inch, and the distance between the external faces of the parapets equal to 14 feet. By similar triangles, $C A H$, $P Q H$;

$$A C : A H :: Q P : Q H;$$

$\therefore Q H = \dfrac{A H \times Q P}{A C} = \dfrac{42 \times 14}{19} = 30 \cdot 9473$ feet, the length of the abutments or springing-lines.

From the chord of the arc equal to 19 feet, and the height of the intrados equal to 7 feet 1 inch, will be found the radius of the circle equal to 9·912 feet, and the length of the arc equal to 25·4026 feet nearly.

Now, $A D^2 = 25 \cdot 4026^2 = 645 \cdot 29208676 = 645 \cdot 3$ nearly.
And $D E^2 = 37 \cdot 4566^2 = 1402 \cdot 99688356 = 1403$ nearly.
$\therefore A E^2 = A D^2 + D E^2 = 2048 \cdot 28897032 = 2048 \cdot 3$ nearly.
$\therefore A E = \sqrt{(2048 \cdot 3)} = 45 \cdot 258$ feet nearly.
$\therefore A E^2 \times r = 2048 \cdot 3 \times 9 \cdot 912 = 20302 \cdot 75$ nearly.
$\therefore R = A E^2 \times r \div A D^2 = 20302 \cdot 75 \div 645 \cdot 3 = 31 \cdot 4$ feet nearly.
$\therefore R' = A E^2 \times r \div D E^2 = 20302 \cdot 75 \div 1403 \cdot = 14 \cdot 4$ feet nearly.

Then, by similar triangles, $A E D$, $F E V$;

$$A E : D E :: F E : E V.$$

$\therefore E V = \dfrac{E D \times F E}{A E} = \dfrac{37 \cdot 4566 \times 30 \cdot 9473}{45 \cdot 258} = 25 \cdot 612$ feet, the breadth of the springing courses on the head of the arch.

$\therefore 45 - 26 = 19$, the number of entire spiral courses, and $45 \cdot 258 - 25 \cdot 612 = 19 \cdot 645$ feet, the length of the soffit of the entire courses.

$\therefore 30 \cdot 9473 \div 26 = 1 \cdot 19$ feet nearly, for the length of the bed of each springer.

$\therefore E u = 25 \cdot 612 \times 12 \div 26 = 11 \cdot 82$ inches, the breadth of the soffit of each springing course.

$\therefore 19 \cdot 643 \times 12 \div 19 = 12 \cdot 406$ inches, the breadths of the soffits of the independent courses.

And, by similar triangles, $E D A$, $E u v$;

$$E D : D A :: E u : u v.$$

$\therefore u v = \dfrac{D A \times E u}{E D} = \dfrac{25 \cdot 4026 \times 11 \cdot 82}{37.4566} = 8 \cdot 001 = 8$ inches nearly, which is the length of the inclined back of each springer.

PLATE 33

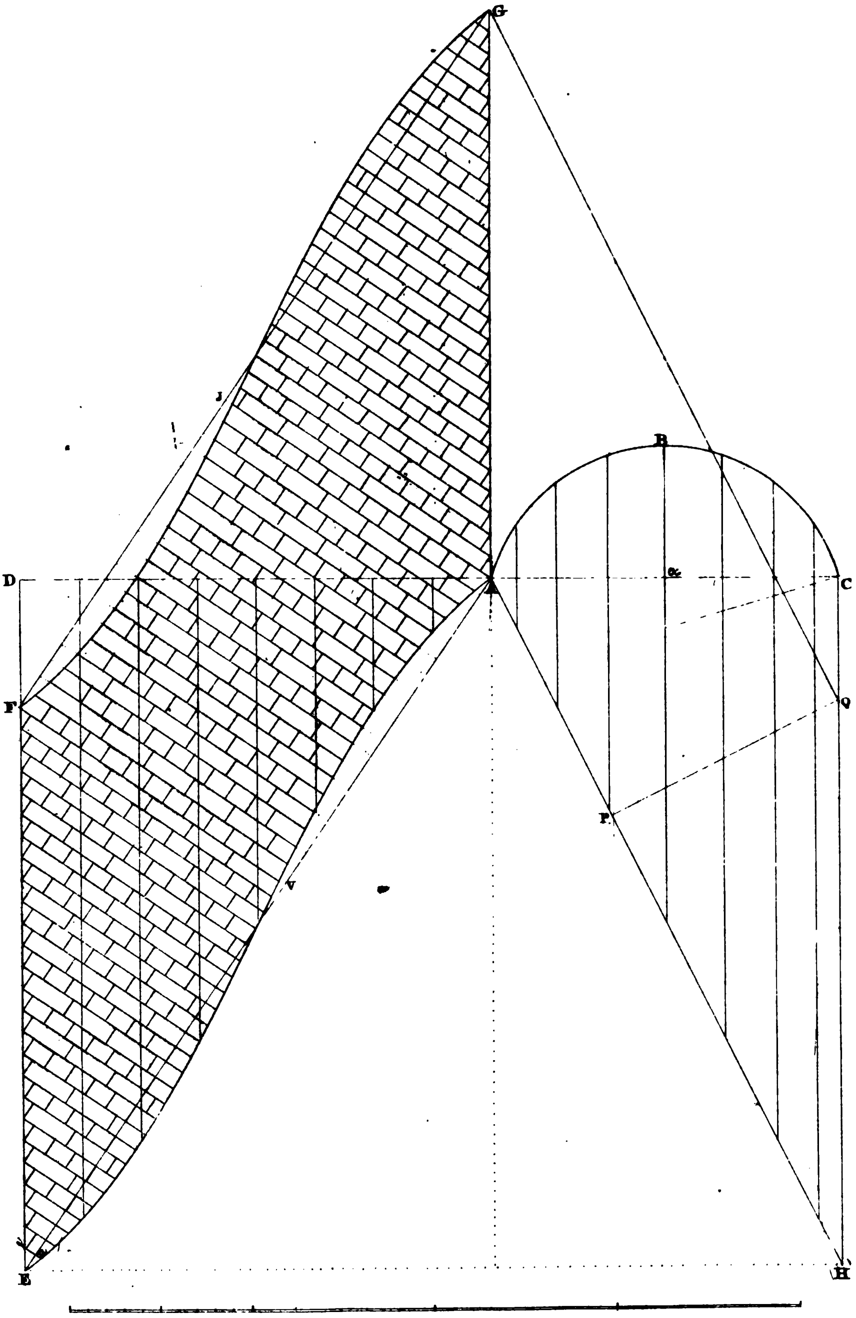
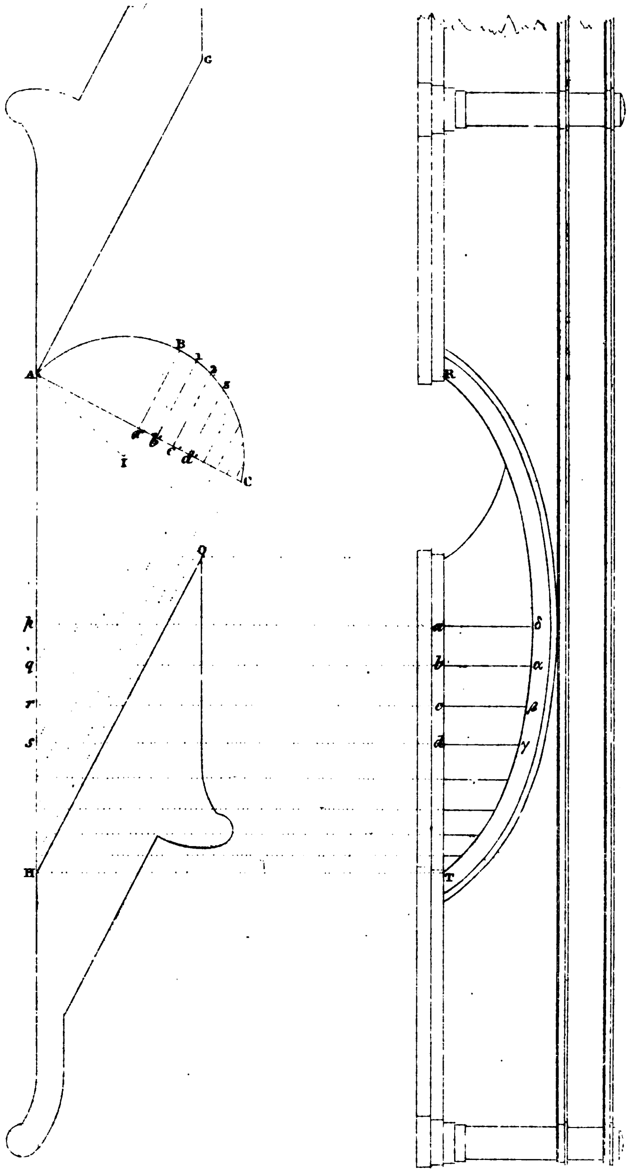


PLATE 34

To face Page 30



Plan and Elevation of the Oblique Bridge over the River Gaunless, near Hagger Leazes Lane, St. Helen's, Auckland.

DESIGNED BY THOMAS STORY, C. E.

This bridge was executed by Thomas Wilson, mason. The manner of tracing the elevation from the plan is as follows:—The right section being *A B C*, and *B* the middle of the arc. In the half-arc *B C*, take the points 1, 2, 3, &c. at pleasure, and draw *B p*, 1 *q*, 2 *r*, 3 *s*, &c. perpendicular to *A C*, meeting *A H* in *p*, *q*, *r*, *s*, and intersecting *A C* in a', b', c', d', &c. Draw *R T* parallel to *A H*, at any convenient distance from *A H*, for the top of the plinth; and draw *p S*, *q* a, *r* β, *s* γ, &c. perpendicular to *A H*, intersecting *R T* in *a*, *b*, *c*, *d*, &c. Make *a S*, *b* a, *c* β, *d* γ, &c. respectively equal to $a'B$, $b'1$, $c'2$, $d'3$, &c. draw the half *S T*, and the other half *R S* being drawn, will form the whole curve *R S T* of the intrados of the arch. The angle of obliquity is computed to be 26° 54′ from the dimensions, which are, the length of each face 42 feet, the width of the arch-way 19 feet, and the distance between the faces 14 feet, as shewn in a drawing sent to me by the engineer, of which the opposite plate is a reduced copy.

OBSERVATIONS.

To construct an oblique arch entirely of stone is, in some countries where it is difficult to procure, very expensive. However, in order to build one which will be sufficiently strong at a moderate price, it is necessary that the imposts or springings should be of stone, and, to have the appearance of good work, the quoins which form the ring-stones and the head of the arch should also be of stone. Then the intermediate parts of the courses may be of brick, (allowing perhaps four courses of bricks to each stone springer,) depending on thickness at the abutment. To work the springers and the quoin-heads, the same templets will be required as if the arch had been constructed entirely of stone. The templets are described in pages 8, 11, 13, 15, 16, 19. Previous to setting the brick courses, the boarding or laggings should be truly adjusted and fixed; and, for the regulation of the work, the bed-lines should be drawn thereon in their true position. In order to try the work as the bricklayer proceeds, he ought to use a kind of set-square, made of thin board, containing an angle exactly the reverse of the templet No. 3, Plate XXI; and, consequently, the curved edge will be concave instead of being convex, as in the arch-square. In trying any course, the set-square must have the point between the curved and straight edge upon the bed-line, the curved edge upon the boarding, and the straight edge upon the side of the course, the two edges being in a plane perpendicular to the bed-line. The sides of each course being made to agree with every application of the set-square, will be what it ought to be. In stone courses, if the stones are truly wrought, the spiral surfaces of the beds will all agree with a set-square; and, therefore, in this case it will be unnecessary to provide one.

A PRACTICAL METHOD

OF CONSTRUCTING AS MUCH OF THE DEVELOPMENT OF THE OBLIQUE ARCH TO THE FULL SIZE, AS WILL BE FOUND SUFFICIENT FOR MAKING THE TEMPLETS FOR THE USE OF THE WORKMEN.

Draw the straight line $O\ C$, (Fig. 1,) and in $O\ C$ take any point E. Draw $E\ A$ perpendicular to $O\ C$, and make $E\ B$ equal to the height of the arch. Prolong $B\ E$ to I, and make $B\ I$ equal to the radius of the cylinder. With the radius $I\ B$ describe the arc $B\ C$, and perpendicular to $E\ C$ draw $C\ F$. Make $C\ F$ equal to the distance of obliquity, and join $E\ F$. Make $E\ O$ equal to the length of the arc $B\ C$, and divide the arc $B\ C$ and the straight line $O\ E$ each into the same number of equal parts, at the points 1, 2, 3, &c. From the points 1, 2, 3, in the arc $A\ C$, draw 1 a', 2 b', 3 c', &c. meeting $E\ C$ in a', b', c', &c. intersecting $E\ F$ in a, b, c, &c.; and from the points 1, 2, 3, &c. in the straight line $O\ E$, draw 1 α, 2 β, 3 γ, &c. Make 1 α, 2 β, 3 γ, &c. respectively equal to $a'\ a$, $b'\ b$, $c'\ c$, &c.; and from O, through the points α, β, γ, &c, draw the curve $O\ \alpha\ \beta\ \gamma\ A$, which will be half of the development of the intradosal line. Join $A\ O$, and $A\ O$ is half the line of the subtense.

The construction of Fig. 1 will become evident by comparing it with Fig. A, which shows the development of the intradosal line to the full extent. Fig. A is a part of the construction of Plate XXIII, of which see the explanation, page 13. The curve-line $O\ \alpha\ \beta\ \gamma\ A$, (Fig. 1,) is identical to the curve-line $o\ \alpha\ \beta\ \gamma\ A$, (Fig. A); and in Fig. A, the part $o\ \alpha\ \beta\ \gamma\ E$ is identical to the part $o\ \alpha\ \beta\ \gamma\ A$; and both parts make the entire development of the intradosal line.

Parallel to $E\ C$, in any convenient place, draw the straight line $a\ e$ (Fig. 2) of any convenient length; and upon $a\ e$ as a chord, with the radius of the cylinder, describe the arc $a\ c\ e$. Draw $a\ f$, $e\ m$, perpendicular to $a\ e$, and draw $e\ f$ perpendicular to $A\ O$. Bisect $a\ e$ in 2, and draw 2–2 parallel to $e\ m$, meeting $e\ f$ in 2, intersecting the arc $a\ c\ e$ in c. From the point 2 in $e\ f$, draw 2 h perpendicular to $e\ f$, and make 2 h equal to 2 c. Through the three points e, h, f, describe the arc $e\ h\ f$. Draw $f\ m$ perpendicular to $e\ f$, and prolong 2–2 to meet $f\ m$ in 2. Draw 2 k perpendicular to $f\ m$; make 2 k equal to 2 c; and through the three points f, k, m, describe the arc $f\ k\ m$.

In any convenient place parallel to $O\ E$ draw $x\ z$, (Fig. 3,) and in $x\ z$ make $x\ y$ equal to half the radius of the cylinder; draw $y\ w$ perpendicular to $A\ O$, (Fig. 1,) and draw $x\ w$ perpendicular to $O\ E$; make $y\ z$ equal to half the breadth of the beds; join $w\ z$; and $y\ w\ z$ is the angle of the twist.

PLATE 35.

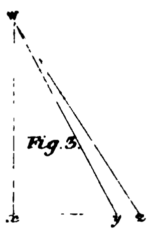


Fig. 3.

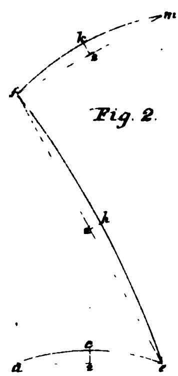


Fig. 2.

Fig. 1.

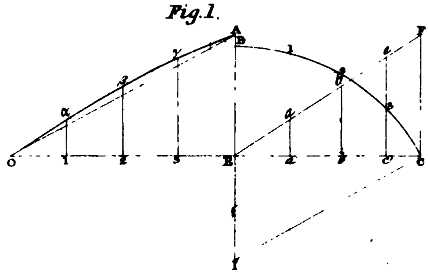
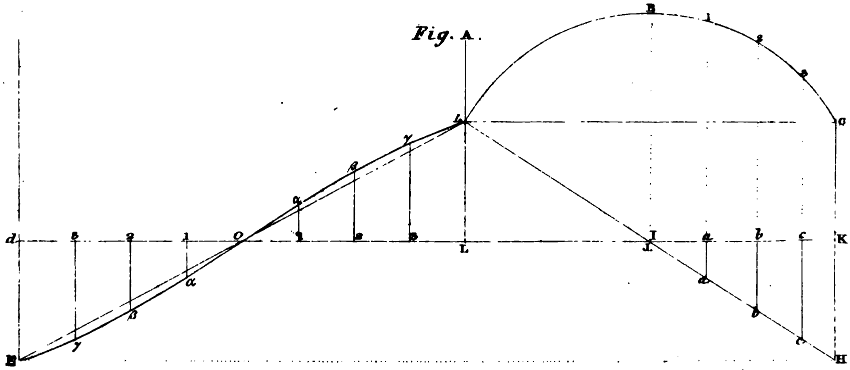


Fig. A.



APPENDIX

CONTAINING VARIOUS ARTICLES CONNECTED WITH THE OBLIQUE ARCH.

ARTICLE I.—To find the solidity of an oblique arch.

RULE.

Multiply the length of the springing-line into the length of the arc of the extrados, and this product into the thickness of the arch or breadth of the courses, and the last product will be the solidity in cubic feet, the dimensions being taken in feet.

The length of the arc of the intrados is generally given, being necessary in the construction; and since similar arcs are to one another as their radii, the length of the arc of the extrados will be easily found.

For it is evident that the development of the convex surface is equal to the parallelogram, of which the base is one of the springing-lines, and the breadth equal to the length of the arc which has the greater radius; and, therefore, the solidity may be found by the rule here given.

EXAMPLE.

Let it be required to find the solidity of the arch, of which the development is shown in Plate XXXII, the length of the springing-line being 37·53 feet, the length of the arc of the intrados 57·32 feet, the radius 25·46, and supposing the thickness of the arch to be 2·5 feet, to find the solidity.

Here $25{\cdot}46+2{\cdot}5=27{\cdot}96$, the radius of the convex surface.

$25{\cdot}46 : 27{\cdot}96 :: 57{\cdot}32 : 62{\cdot}94$, the length of the arc of extrados.

$\therefore\ 37{\cdot}53\times 62{\cdot}94\times 2{\cdot}5=5905{\cdot}3455$ feet, the solidity required.

This measure, on account of the waste of stone, will not be more than sufficient for the arch-stones alone; and if the waste in forming the springers be considered, the quantity cut away would amount to a course of stone equal to the length of the abutment, which ought, therefore to be added to the solidity of the arch.

ARTICLE II.—To find the angles for executing an occupation arch with plane joints.

An occupation arch is an arch perforating the mound or bank of earth raised to carry a railway, the mound being sustained on each side by a wall which terminates in a battering face.

Let the semi-circle *A B C* be a right section of the arch, *E* being the centre, and *A C* the diameter.

Draw *E B* perpendicular to *A C*, and draw *E D*, making the angle *B E D* equal to the quantity of batter. Divide the arc *A B C* into as many equal parts as there are arch-stones, say nine, at the points 1, 2, 3, &c.; and parallel to *A C* draw the lines 1–1′, 2–2′, 3–3′, &c. intersecting *E D* in *f*, *g*, *h*, &c. and meeting the opposite side of the semi-circular arc at the points 1′, 2′, 3′, &c. Prolong *B E* to any convenient point *G*, and through *G* draw *P R* parallel to *A C*. In *G E* make *G k*, *G l*, *G m*, &c. respectively equal to *E f*, *E g*, *E h*, &c.; and through the points *k*, *l*, *m*, &c. parallel to *P R*, draw *p p′*, *q q′*, *r r′*, &c. Parallel to *B G* draw *A P*, 1 *p*, 2 *q*, 3 *r*, &c. as also *C R* 1′ *p′*, 2′ *q′*, 3′ *r′*, &c. Through the points *p*, *q*, *r*, &c. draw the curve *P p q r s s′ r′ q′ p′ R*, and from *G* through the points *p*, *q*, *r*, &c. draw the straight lines *G p α*, *G q β*, *G r γ*, &c.; and the parts *p α*, *q β*, *r γ*, &c. are the joint-lines on the battering face.

In *G P* take *G L* of any convenient length, and draw *L H*, *L I*, *L J*, &c. respectively perpendicular to *G s*, *G r*, *G q*, &c. meeting *G s*, *G r*, *G q*, in *H*, *I*, *J*, &c.; make the angle *G L M* equal to the angle *B D E*, and draw *G M* perpendicular to *G L*; in *G L* make *G a*, *G b*, *G c*, &c. respectively equal to *G H*, *G I*, *G J*, &c.; join *a M*, *b M*, *c M*, &c.; and the angles *G d M*, *G c M*, *G b M*, &c. or the angles *R d M*, *R c M*, *R b M*, &c. are the dihedral angles made by the planes of the beds, and the planes of the faces of the arch, at the joints *p α*, *q β*, *r γ*.

In *L P* make *L t*, *L u*, *L v*, *L w*, respectively equal to *L H*, *L I*, *L J*, *L K*; from *L*, with the radius *L M*, describe the arc *M z′ z y x*; draw *t x*, *u y*, *v z*, *w z′*, perpendicular to *G P*; join *L x*, *L y*, *L z*, *L z′*; and the angles *P L z′*, *P L z*, *P L y*, *P L x*, are the angles made by the joint-lines of the soffit and the joint-lines of the face of the arch, or what has been called the angles of the beds.

The same angles are also to be used for the other half of the arch.

The reader will here recognise the principles of the trihedral, as shown on page 43, Plate XL. The principle of this arch is the same as that of an oblique arch, the angle of obliquity being in a vertical instead of a horizontal plan.

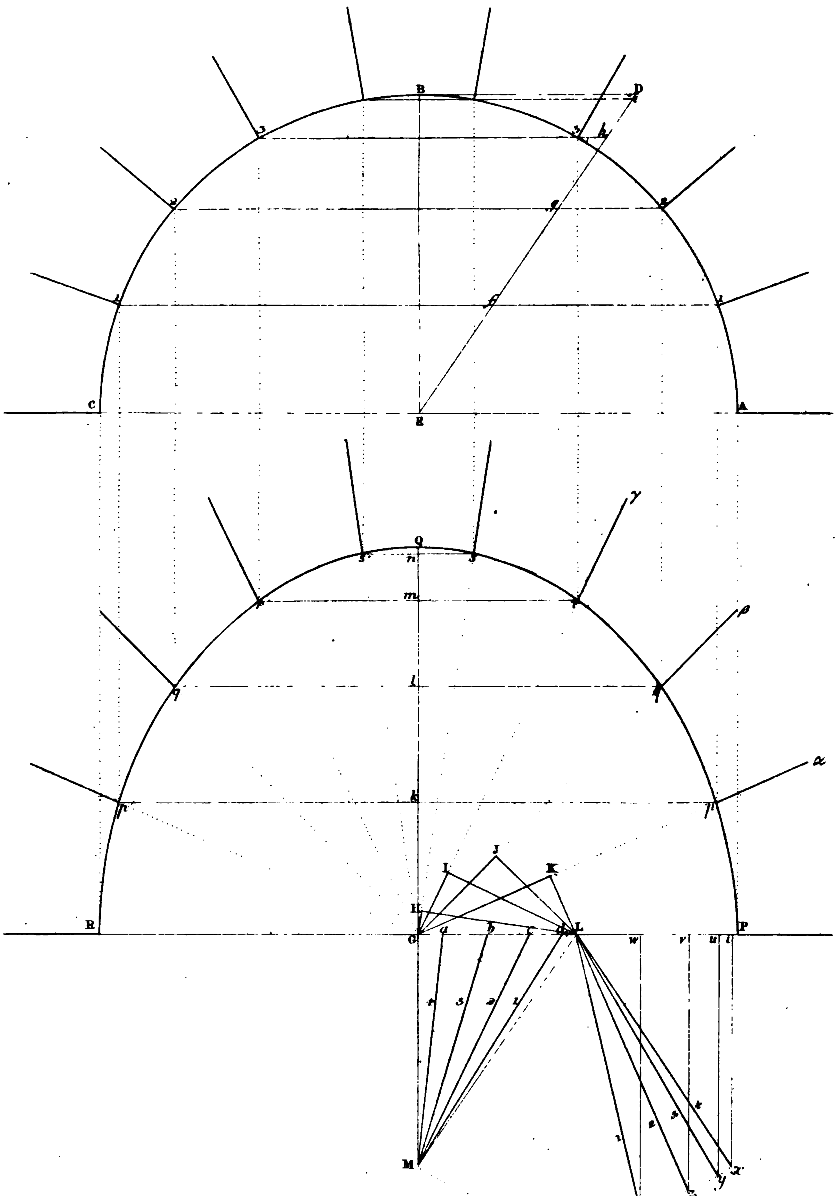

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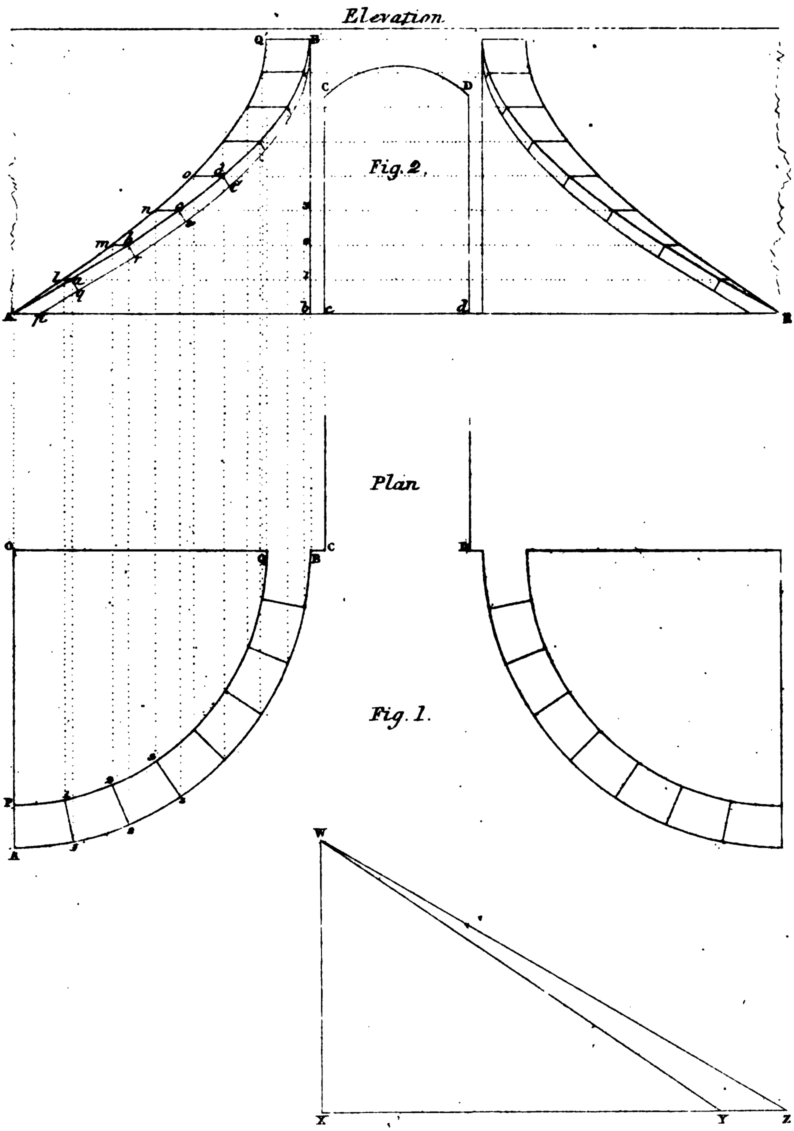
PLATE 37



ARTICLE III.—To draw the plan and elevation of an occupation arch, supposing the ground on one side of it to have a considerable ascent, and the earth before it to be retained by a wall on each side of the approach, each wall being built upon a circular plane, and terminated in a regular spiral surface, meeting the face of the arch in its highest point of elevation.

Let $A\ B\ Q\ P$ (Fig. 1) be one side of the plane, $A\ B$ the line of the outer face, and $P\ Q$ that of the inner face, $A\ B$ and $P\ Q$ being arcs of circles described from the centre O; therefore $A\ P$ or $B\ Q$ is the thickness of the coping.

Divide the arc $A\ B$ into any convenient number of equal parts, at the points 1, 2, 3, &c. and draw the lines 1–1′, 2–2′, 3–3′, &c. radiating to the centre O, and meeting the arc $P\ Q$ in the points 1′, 2′, 3′, &c.

In the elevation, (Fig. 2,) let the ground-line $A'\ R'$ be parallel to $O\ B$ on the plan; perpendicular to $A'\ R'$, draw $A\ A'$, $B\ B'$, $C\ C'$, $D\ D'$, intersecting $A'\ R'$ in $A'\ b\ c\ d$; make $b\ B'$ equal to the height of the coping, and make $c\ C'$, $d\ D$, each equal to the height of the arch-way; describe the segment $C''\ D'$ to the height of the intrados; and $c\ C''$, $D'\ d$, is the elevation of the arch-way or aperture. Divide $b\ B'$ at the points 1, 2, 3, &c. into the same number of parts into which the arc $A\ B$ is divided. Parallel to the ground-line $A'\ R'$ draw 1 l, 2 m, 3 n, &c.; and from the points 1, 2, 3, &c. in the arc $A\ B$, draw lines 1 a, 2 b, 3 c, &c. perpendicular to the ground-line $A'\ R'$, meeting the lines 1 l, 2 m, 3 n, &c. in the points a, b, c, &c. Also perpendicular to $A'\ R'$, draw 1′ l, 2′ m, 3′ n, &c. From A' through the points a, b, c, &c. draw the curve $A\ a\ b\ c \ldots B'$; also from A', through the points l, m, n, &c. draw the curve $A\ l\ m\ n \ldots Q$. The curve-lines $A'\ B'$, $A'\ Q'$, are the projections of the spiral-lines which contain the spiral surface of the coping. The curve-line $p\ q\ r\ s \ldots$ is the projection of the spiral of the bed of the stone which shows in the cylindric surface of the wall; and thus the two spiral-lines, represented by $A'\ a\ b\ c \ldots B'$ and $p\ q\ r\ s \ldots$, contain that portion of the convex surface of the cylinder which is required to be formed on the edge of the coping.

If the centre of the circular arcs $A\ B$, $P\ Q$, be in the line $O\ B$, which is the wall-line of the face of the arch, the spiral surface terminating the top of the wall will meet the plane of the face of the arch in a straight line parallel to the horizon; but if the centre is not in the wall-line of the face of the arch, the intersection of the spiral surface and the plane of the face of the wall will not be a straight line, but a curve inclined in all points to the horizon. In

either case, however, the method of forming the copings will be the same. It is a matter of minor importance whether the line in which the spiral surface meets the plane of the face of the arch in a straight or curved line, or whether the line of meeting be parallel or inclined to the horizon, depending entirely on the taste of the workman or inspector. If the section which would fit upon the face of the arch be inclined, it may be brought to a level by cutting off the projecting point by a plane parallel to the horizon.

Draw the straight line *X Z*, and make *X Z* equal to the length of the arc *A B;* and in *X Z* make *X Y* equal to the length of the arc *P Q;* draw *X W* perpendicular to *X Z;* join *W Y* and *W Z;* and *Y W Z* is the angle of the twist.

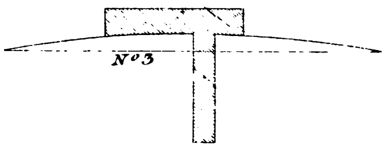
ARTICLE IV.—To form the coping-stones for the circular wing-walls upon the right and left. (See the plan preceding the plate.)

Having found the angle of the twist *Y W Z*, as shown in the said plate, draw the straight line *a e* of any convenient length; and with the chord *a e*, and the radius *O A* or *O B*, (Fig. 1, Plate XXXVII,) describe the arc *a c e*. Draw *a f* perpendicular to *a e*, and make the angle *a e f* equal to the angle *X Y W* (Fig. 3, Plate XXXVII). Draw *e m* perpendicular to *a e*, and *f m* perpendicular to *f e*. Bisect *a e*, *e f*, *f m*, each in the point 2. Draw 2 *c* perpendicular to *a e*, 2 *h* perpendicular to *e f*, and 2 *k* perpendicular to *f m*. Make 2 *h*, 2 *k*, each equal to 2 *c*, and describe the arcs *e h f*, *f k m*.

Then the bed of the stone being formed to the twist or spiral surface, as in the oblique arch, the narrow face, which is the edge of the stone, will be formed to the cylindric surface of the wall by the arch-square No. 3, observing that as the face of the wall corresponding to the arc *A B* (Fig. 1, Plate XXXVII) is convex, the curved limb of the inner edge of the arch-square must be concave.

The bed and upper surface of the coping for the left-hand cylindric wall are right-hand spiral surfaces, and the bed and upper surface of the coping of the right-hand cylindric wall are left-hand spiral surfaces; for the plan of the wall on the left hand is the same as the plan of a right-hand spiral stair, and the plan of the wall on the right hand the same as a left-hand spiral stair.

PLATE 38



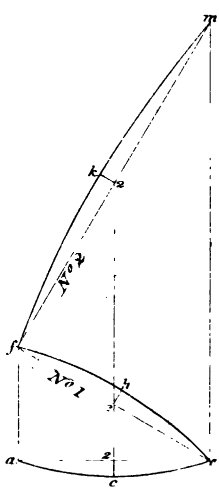
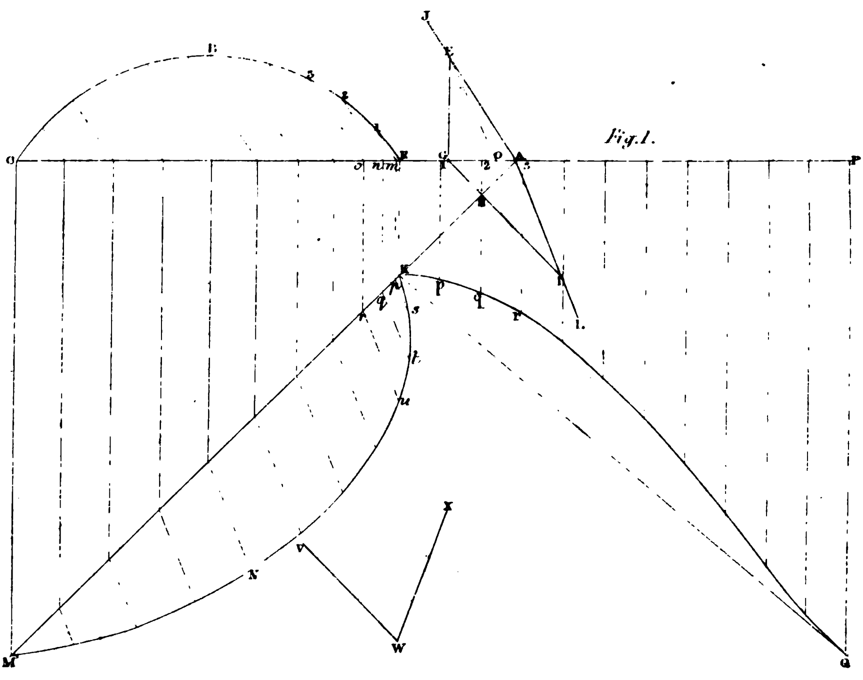


PLATE 39



ARTICLE V.—To construct an oblique arch in a battering wall.

Let *C M K* (Fig. 1) be the angle of obliquity, *F B C* the right section of the arch, and *V W X* the complement of the angle of the batter of the wall, or the dihedral angle which the face of the wall makes with the level or horizontal plane of the base. Prolong *C F* and *M K* to meet each other in *A*.

Then in a trihedral are given the angle *F A K* of the adjacent face, and the dihedral angle adjacent, to find the angle of the opposite face, and the angle of the oblique face.

In *A F* take any convenient point *G*, and draw *G I* intersecting *A K* perpendicularly in *H*. "In *A F* make *G O* equal to *G H*, and make the angle *G O E* equal to the given dihedral angle v w x; draw *E* perpendicular to *A G*, and through *E* draw *A J*; and the angle *F A J* is the angle of the opposite face."

"From *A*, with the distance *A E*, cut *H I* in *I*, and through *I* draw *A L*; and the angle *K A L* is the angle of the oblique face," as in Prob. XXIII, Introduction.

Prolong *C F* to *P*, and make *F P* equal to the length of the arc *F B C*. Divide the arc *F B C* and the straight line *F P* each into an equal number of equal parts, at the points 1, 2, 3, &c. From the points 1, 2, 3, &c. in the arc *F B C*, draw 1 *m*, 2 *n*, 3 *o*, &c. parallel to *A J*, meeting *E C* in the points *m*, *n*, *o*, &c. Parallel to *C M* draw *F K*, *m p*, *n q*, *o r*, &c. meeting *K M* in *K*, *p*, *q*, *r*, &c. Parallel to *A L*, draw *p s*, *q t*, *r u*, &c.; and make *p s*, *q t*, *r u*, &c. respectively equal to *m* 1, *n* 2, *o* 3, &c.; and through the points *s*, *t*, *u*, &c. draw the curve *K N M*, which is the section of the cylinder, or the under edge of the elevation of the oblique face of the arch.

Through the points 1, 2, 3, &c. in the straight line *E P* parallel to *C M*, draw 1 p, 2 q, 3 r, &c; make 1 p, 2 q, 3 r, &c. respectively equal to *m p*, *n q*, *o r*, &c.; through the points *K*, p, q, r, &c. draw a curve; and the curve *K* p q r...*Q* shall be the development of the oblique section *K N M*.

Draw the straight line *K Q*, and having divided it into as many equal parts as the courses are in number, and having the length of the springing-line which is parallel to *C M* given, the development of the intrados, and all the necessary templets and angles for working the stones, may be found by proceeding in the usual manner as when the two faces are vertical and in parallel planes.

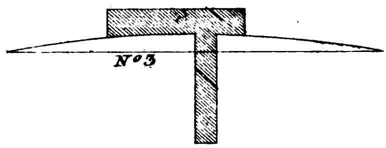
But in the construction of an oblique arch in a battering wall, the curve-line of the development of the arch differs more from its line of subtense *K Q*, than when the planes of the elevation are vertical and parallel; and more particularly so when, in the oblique arch with vertical faces, the less part of the arc of the right section is of the entire circle. Oblique arches which have battering faces are therefore not so strong as those which have their faces in parallel planes.

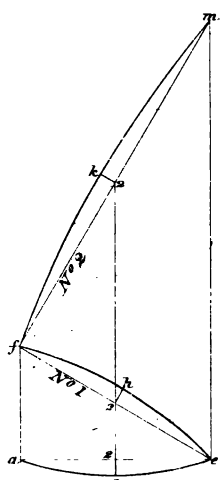
OBLIQUE ARCH WITH PLANE JOINTS.

An oblique arch with plane joints is that in which the beds of the stones are planes, passing through the axis of the cylinder.

In the oblique arch with plane joints, the planes of the joints being parallel to the axis, intersect each face of the arch in very oblique angles, and only one of the joints can be perpendicular to the face, and this is when each elevation is a semi-ellipse, and when the plane of a joint intersects the figure in a straight line coinciding with the axis-major. All the other joints, as they recede from the centre, are more and more oblique till they reach the summit of the arch. As every oblique joint causes the dihedral angles made by the face and that joint to be very unequal, the obtuse dihedral angle will be much stronger than that which is acute, these angles being supplements of each other. Therefore oblique arches with plane joints should never be used where great strength is necessary; and where the angle of obliquity is very acute, the oblique arch with spiral joints should only be employed, as the spiral joints are as nearly perpendicular to the face as the construction will admit of. For in order to have the stones of one identical form, or such as would fill the same mould were they of one length, it is necessary that the development of the spirals which form the bed-lines should be parallel straight lines, of which two of them will be perpendicular to the face of the arch, and the others nearly so.

The cylinder here referred to is not that of a simple cylinder, but of that description which mathematicians call a hollow cylinder, consisting of two concentric surfaces, of which the interior is concave, and the exterior convex. These surfaces, without regarding the distinction of concave and convex, are called by the name of cylindric surfaces.


No 3


m
k
2
No 2
f
h
No 1
3
a
2
e
c

Given the elevation and plan of a semi-elliptic arch, which is the oblique section of a cylinder, of which the right section of the intrados is a semi-circle, to find the angles for cutting the quoin-heads.

Let No. 1 be a plan of the arch, of which the angle *U V W* is that of the acute-angled abutment; and let No. 2 be the elevation, in which the lines *b h*, *c i*, *d j*, &c. represent the joints of the stones in the face, meeting the inner curve in the points *b*, *c*, *d*, &c. and the outer curve in the points *h*, *i*, *j*, &c.

Draw $\alpha\,\beta$ parallel to *V Y*, at such a distance from *V Y* as the plane of the rear elevation may be supposed to be distant from the plane of the front, meeting *V W* in α and *Y Z* in β; and the parallelogram α *V Y* β shall be the plan of the aperture. Draw *V* γ perpendicular to *V Y*, meeting $\alpha\,\beta$ in γ. Draw *b g*, *c l*, *d n*, &c. parallel to *U V* or *g u*; make *b g*, *c l*, *d n*, &c. each equal to $\alpha\,\gamma$, and complete the parallelograms *g b h l*, *l c i m*, *n d j o*, &c. which shall represent the beds of the stones.

Now, observing that each of the parallel lines *b g*, *c l*, *d n*, &c. is the projection of a joint-line in the intrados of the arch, and forms an angle with the face of the arch in a plane perpendicular to that face, equal to the angle *U V W* of the acute abutment, it is therefore evident that at every joint will be formed a right trihedral, of which the angle of one of the right faces is equal to the acute angle of the plan, and the other in the plane of the face equal to the angle made by the joint-line and a line parallel to the base, and that the angle of the oblique face is the angle made by the two joint-lines, the one being in the face and the other in the intrados.

To construct the trihedral for any one of the joints, as *b h* in No. 3 or in No. 4, draw *A K* parallel to *b h*, and *A F* parallel to *b g*; and make the angle *F A J* equal to the angle *U V W*. Then *F A K* is the angle of the adjacent face, and *F A J* the angle of the opposite face. Proceed now as in Prob. XXI, page xxxiv, thus:—

From any convenient point *G*, in the right edge *A F*, draw *G I*, intersecting the adjacent edge *A K* perpendicularly in *H*, and draw *G E* perpendicular to *A F*, meeting *A J* in *E*. From the point *A*, with the distance *A E*, cut *H I* in *I*; through *I* draw *A L*; and the angle *K A L* or *H A I* shall be the angle of the oblique face, that is, the angle made by the joint-line on the intrados, and the joint-line on the face.

In *A F* make *G O* equal to *G H*; join *O E*; and the angle *G O E* is the dihedral angle, or the angle made by the bed and face of the stone.

Or the angles of the beds, and the dihedral angles of the faces and beds, may be found in one diagram, in the following manner :—

In Fig. 2 draw $P\ Q$ parallel to $A\ F$ or $V\ Y$, and in $P\ Q$ take the point L, and draw $L\ H$, $L\ I$, $L\ J$, &c. respectively parallel to the joint-lines $b\ h$, $c\ i$, $d\ j$, &c. in the elevation No. 2; and towards P take the distance $L\ G$ of any convenient length, and draw $G\ H$, $G\ I$, $G\ J$, &c. respectively perpendicular to $L\ H$, $L\ I$, $L\ J$, &c. In $G\ L$ make $G\ a$, $G\ b$, $G\ c$, &c. respectively equal to $G\ H$, $G\ I$, $G\ J$. Make the angle $G\ L\ M$ equal to the acute angle which the axis of the cylinder makes with the line of section of the elevation, or equal to the angle of the acute abutment, viz. equal to $U\ V\ W$, and draw $G\ M$ perpendicular to $P\ Q$. Join $a\ M$, $b\ M$, $c\ M$, &c.; and the angles $P\ a\ M$, $P\ b\ M$, $P\ c\ M$, &c. shall be the dihedral angles made by the beds and faces, at the joints $b\ h$, $c\ i$, $d\ j$, &c. From L, with the radius $L\ M$, describe the arc $M\ z\ y\ x$. In $L\ Q$ make $L\ u$, $L\ v$, $L\ w$, &c. respectively equal to $L\ H$, $L\ I$, $L\ J$, &c. Perpendicular to $P\ Q$ draw $u\ x$, $v\ y$, $w\ z$, &c.; and the angles $Q\ L\ x$, $Q\ L\ y$, $Q\ L\ z$, &c. are the angles of the beds. To avoid confusion, the work is placed below the line $P\ Q$, instead of being above.

REMARKS.

In the construction of the oblique arch with plane joints, the arch-stones which are supported by the acute-angled abutment have their dihedral angles adjacent to the upper beds as they rise towards the middle decreasing. The first stone has its dihedral angle equal to a right angle; and if a stone be supposed to have a joint in the summit of the arch, the dihedral angle of that stone would be equal to the acute angle of the abutment. Also, the angles of the beds of the stones which rise from the said acute-angled abutment are, on the contrary, continually increasing as they ascend towards the crown, the lower bed of the first stone being the same as the acute-angled abutment upon which it is placed; and if a joint be supposed to be at the summit, the angle of the bed of the adjacent stone would be a right angle.

With regard to the dihedral angles of the arch-stones which rise from the obtuse-angled abutment, those of the upper beds are continually increasing as they ascend towards the summit or crown, the dihedral angle of the first stone at the bottom bed being a right angle, and that at the summit equal to the angle of the obtuse abutment. Also, the angles of the beds as they rise towards the crown are, on the contrary, continually decreasing from an angle equal to that of the abutment, to a right angle at the crown.

PLATE. 41

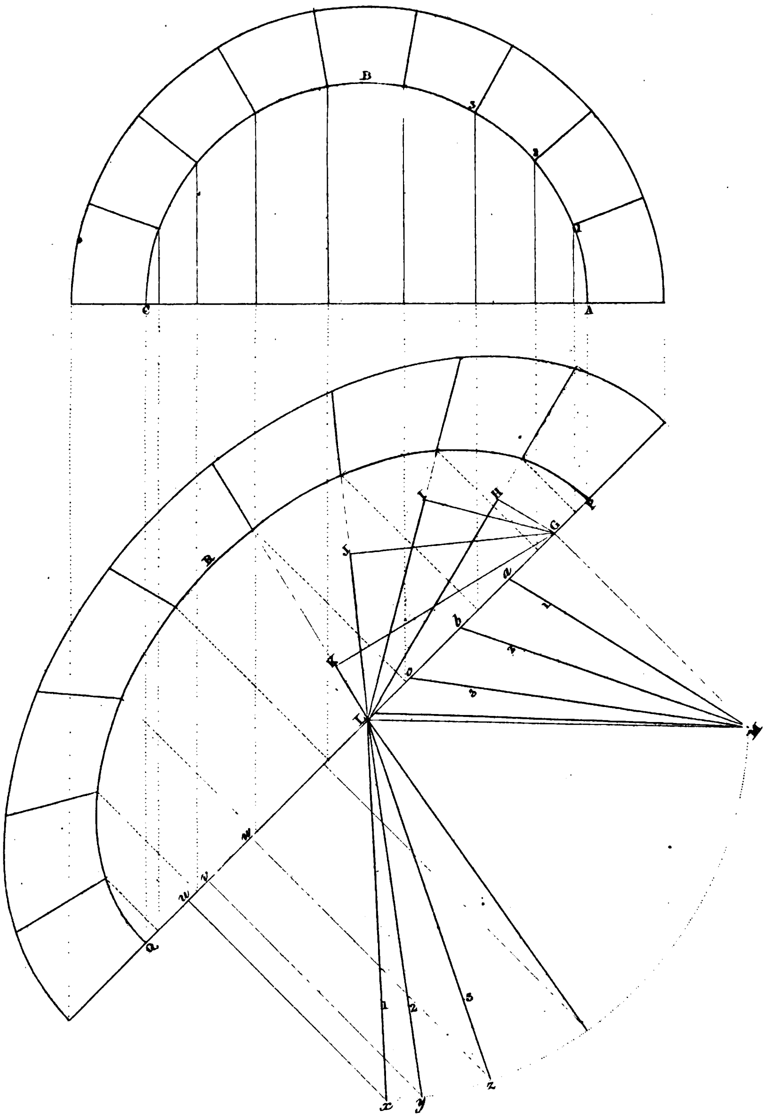
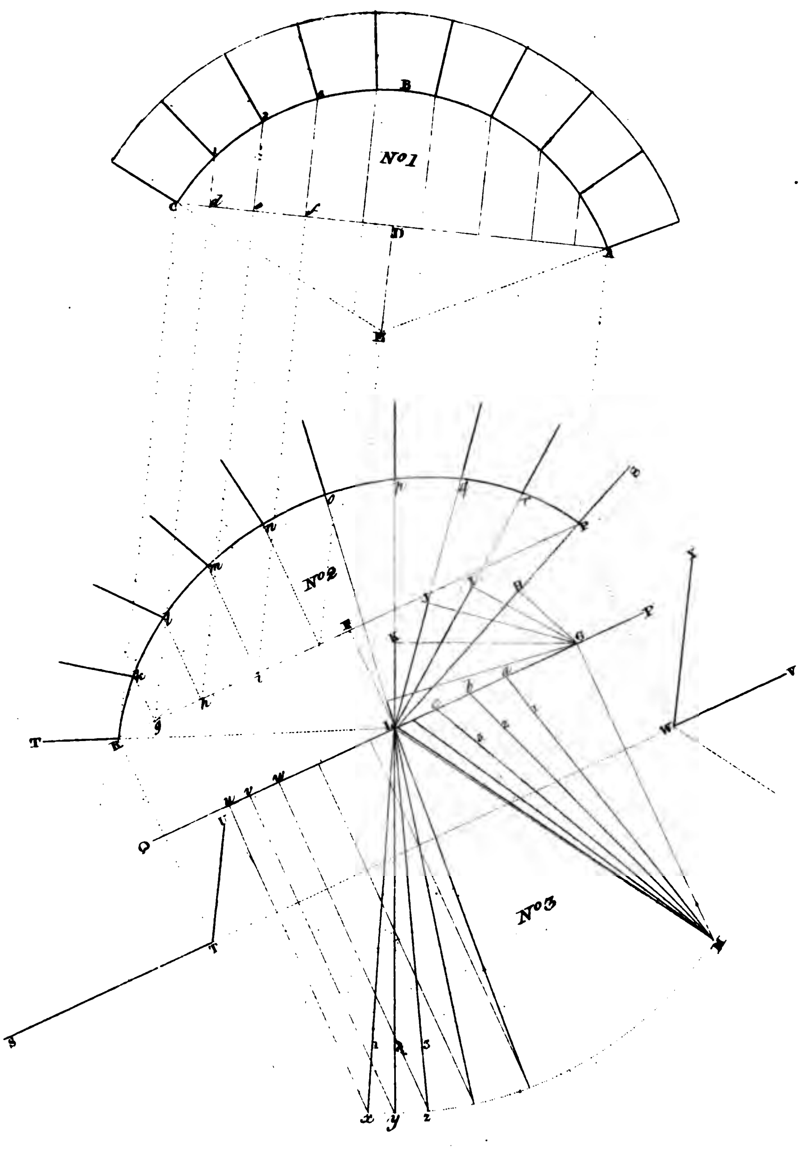


PLATE 42



On both sides of the arch the dihedral angles of the under beds are the supplements of those of the upper beds, and the angles of the beds which rise from the obtuse-angled abutment are the supplements of the angles of the beds of the stones which rise from the acute-angled abutment.

The position of the joints in Plate XL is not confined. In Plates XLI and XLII they are regulated by joints of the right section, and are found as in Prob. XIV, page xxi, Introduction. In Plates XLI and XLII, *L H*, *L I*, *L J*, &c. are the continuations of the joints to the centre *L*. In Plate XLII the elevation answers to a right section, which is the segment of a circle.

The method of finding the dihedral angles, as also the angles of the beds, will be found as before explained, thus:—

Draw *G H*, *G I*, *G J*, (Plate XLII,) respectively perpendicular to *L H*, *L I*, *L J*, &c. In *G L* make *G a*, *G b*, *G c*, &c. respectively equal to *G H*, *G I*, *G J*, &c. Make the angle *G L M* equal to the acute angle which the axis of the cylinder makes with the line of the elevation, or equal to the angle of the acute abutment, viz. equal to *V W X*, and draw *G M* perpendicular to *G L*. Join *a M*, *b M*, *c M*, &c.; and the angles *P a M*, *P b M*, *P c M*, &c. shall be the dihedral angles at the joints which radiate to the centre *L*. From *L*, with the radius *L M*, describe the arc *M z y x*. In *L Q* make *L u*, *L v*, *L w*, &c. respectively equal to *L H*, *L I*, *L J*, &c. Perpendicular to *G L* draw *u x*, *v y*, *w z*, &c.; and *Q L x*, *Q L y*, *Q L z*, &c. are the angles of the beds.

TESTIMONIALS

REGARDING THE SUCCESS WHICH THE AUTHOR HAS HAD IN THE APPLICATION OF HIS PRINCIPLES TO THE EXECUTION OF OBLIQUE ARCHES.

THE first is a letter from Mr. JAMES HOGG to HENRY WELCH, Esq. Bridge Surveyor to the County of Northumberland, of which the following is a copy:—

"SIR—Having constructed an oblique arch for the Messrs. Robsons, builders, of Newcastle-upon-Tyne, upon the principles laid down in "Nicholson's Treatise on Stone-cutting," I have no hesitation in saying that it is perfectly practicable to dress the arch-stones at the quarry, before bringing them to the intended spot, correctly, and in every way according to Mr. Nicholson's rule, which, in fact, was my sole guide in the bridge alluded to.

"JAMES HOGG, *Operative Mason*."

"*Newcastle-on-Tyne, June* 21, 1836."

This arch was executed on the Hartlepool Railway, near Castle Eden, in the year 1834.

Most of the oblique arches in the counties of Northumberland and Durham have been constructed according to the principles shown in the "Guide to Railway Masonry," particularly those upon the Newcastle and North Shields Railway. The author was therefore anxious to ascertain how they have stood, and what difficulties the workmen had found in their execution. Having applied to the Engineer upon the line, he was so obliging as to return the following report:—

"*Royal Arcade, Newcastle-upon-Tyne, July* 29, 1839.

"DEAR SIR—In reply to your inquiry, I beg to state that I have built nine oblique stone bridges upon the Newcastle and North Shields Railway according to your principle, all of which have stood extremely well. The workmen found no difficulty in dressing or setting the stones correctly, after the principle was explained to them.

"I remain, dear Sir, yours most truly,

"ROBT. NICHOLSON."

"To PETER NICHOLSON, Esq. Newcastle-upon-Tyne."

From Mr. JOHN BATEY and Mr. JOHN RIDLEY to Mr. PETER NICHOLSON.

"SIR—Having constructed five oblique arches for Messrs. Grahamsley and Reed, Contractors, upon the Brandling Junction Railway, according to the principles laid down by you in your work on the "Construction of the Oblique Arch," we have no hesitation in saying you have rendered them quite easy to be understood by the workmen, and particularly the method for finding the bevels for working the face-stones of the arch, which we have done so as to need no paring after they were set.

"We are, Sir, your most obedient servants,

"JOHN BATEY, *Foreman of the Masons.*

"JOHN RIDLEY, *Inspector.*"

"To PETER NICHOLSON, Esq. Newcastle-upon-Tyne."

"*Elswick Villas, Newcastle, April* 18, 1840.

"SIR—Having now seen my design of the oblique bridge over the river Tees, on the line of the Great North of England Railway, successfully carried into effect, I am enabled to speak with certainty upon the correctness of your published principles for the construction of oblique arches. My bridge consists of four arches, built at an angle of 50°. The chord of the right section of each arch is 45·96 feet, and that of the oblique section 60 feet. I may also state, that I consider your work on the Oblique Arch the most practically useful of any that I have seen; and as the structure which is near to Croft fully warrants the highest opinion of it, I beg, as a member of the profession for which you have done much, to thank you for the great pains you have taken in working out so clearly the principles of the oblique arch.

"I am, Sir, your most obedient servant,

"HENRY WELCH, *Civil Engineer.*"

"To PETER NICHOLSON, Esq."

The following paragraph on the subject of this bridge is extracted from the "Newcastle Courant," of April 24, 1840:—

"GREAT NORTH OF ENGLAND RAILWAY.—The key-stone of the last unfinished arch of the Great North of England Railway Company's bridge over the river Tees, at Croft, was fixed in its place on Thursday, the 16th instant, by George Hutton Wilkinson, Esq., of Harperly Park, the Chairman of the Board of Directors, in the presence of several of the directors of the Company, and a numerous body of

spectators. The foundation-stone of the bridge was laid by Mrs. Wilkinson, the lady of the chairman, in the month of May, 1838, and the bridge is now rapidly approaching its completion. Mr. Wilkinson, after laying the stone, made an able address to the bystanders, and observed that the completion of the bridge was an earnest to the shareholders and the public of the speedy completion of the great national undertaking of which it formed a part. The directors, engineers, and other officers of the company present at the ceremony, were afterwards entertained at the Spa Hotel, Croft, by Mr. Welch, of Newcastle, the talented engineer of the bridge. Among the company present was Mr. Peter Nicholson, the discoverer of the spiral principle on which the skew bridge was built, who expressed himself much gratified by the successful manner in which his theory had been carried into practice, this bridge being of great magnitude, and the eskew at an angle of 50°."

"*West Street, Arthur's Hill, Newcastle, May* 26, 1840.

"SIR—As one of the contractors (with William and James Dees) for the Tees Bridge, near Croft, on the Great North of England Railway, I beg leave to make the following statements regarding your "Guide to Railway Masonry," namely, that it is by far the most practically useful and correct method that I have as yet seen laid before the public for the construction of an oblique arch; that there was not the least difficulty in finding the templets necessary, as laid down by you, for working the arch-stones of this bridge; and that, when explained to the workmen, they found no difficulty whatever in working the same. In fact, your method renders the working of them nearly as simple as those of a common square arch. The Tees Bridge consists of four arches, built at an angle of 50°, the oblique section being 60 feet, and the height of the arches 14·5 feet. When the centres of this bridge were removed, the crown of the arches was found not to descend more than three-quarters of an inch in any one of them, which is even less than has been found in the construction of many common square arches of the same magnitude. In conclusion, I beg leave to thank you for the great pains you have taken in rendering the oblique arch practically easy to most capacities.

"I am, Sir, your most obedient servant,

"JAMES HOGG."

"To PETER NICHOLSON, Esq."

TABLE

OF

NATURAL SINES,

FOR THE USE

OF THE

PRECEDING TREATISE ON THE OBLIQUE ARCH.

SEE SECTION V, PAGE XLIX.—INTRODUCTION.

	0 Deg.		1 Deg.		2 Deg.		3 Deg.		4 Deg.		5 Deg.		
M.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	
0	0	100000	1745	99985	3490	99939	5234	99863	6976	99756	8716	99619	60
1	29	100000	1774	99984	3519	99938	5263	99861	7005	99754	8745	99617	59
2	58	100000	1803	99984	3548	99937	5292	99860	7034	99752	8774	99614	58
3	87	100000	1832	99983	3577	99936	5321	99858	7063	99750	8803	99612	57
4	116	100000	1862	99983	3606	99935	5350	99857	7092	99748	8831	99609	56
5	145	100000	1891	99982	3635	99934	5379	99855	7121	99746	8860	99607	55
6	175	100000	1920	99982	3664	99933	5408	99854	7150	99744	8889	99604	54
7	204	100000	1949	99981	3693	99932	5437	99852	7179	99742	8918	99602	53
8	233	100000	1978	99980	3723	99931	5466	99851	7208	99740	8947	99599	52
9	262	100000	2007	99980	3752	99930	5495	99849	7237	99738	8976	99596	51
10	291	100000	2036	99979	3781	99929	5524	99847	7266	99736	9005	99594	50
11	320	99999	2065	99979	3810	99927	5553	99846	7295	99734	9034	99591	49
12	349	99999	2094	99978	3839	99926	5582	99844	7324	99731	9063	99588	48
13	378	99999	2123	99977	3868	99925	5611	99842	7353	99729	9092	99586	47
14	407	99999	2152	99977	3897	99924	5640	99841	7382	99727	9121	99583	46
15	436	99999	2181	99976	3926	99923	5669	99839	7411	99725	9150	99580	45
16	465	99999	2211	99976	3955	99922	5698	99838	7440	99723	9179	99578	44
17	495	99999	2240	99975	3984	99921	5727	99836	7469	99721	9208	99575	43
18	524	99999	2269	99974	4013	99919	5756	99834	7498	99719	9237	99572	42
19	553	99998	2298	99974	4042	99918	5785	99833	7527	99716	9266	99570	41
20	582	99998	2327	99973	4071	99917	5814	99831	7556	99714	9295	99567	40
21	611	99998	2356	99978	4100	99916	5844	99829	7585	99712	9324	99564	39
22	640	99998	2385	99972	4129	99915	5873	99827	7614	99710	9353	99562	38
23	669	99998	2414	99971	4159	99913	5902	99826	7643	99708	9382	99559	37
24	698	99998	2443	99970	4188	99912	5931	99824	7672	99705	9411	99556	36
25	727	99997	2472	99969	4217	99911	5960	99822	7701	99703	9440	99553	35
26	756	99997	2501	99969	4246	99910	5989	99821	7730	99701	9469	99551	34
27	785	99997	2530	99968	4275	99909	6018	99819	7759	99699	9498	99548	33
28	814	99997	2560	99967	4304	99907	6047	99817	7788	99696	9527	99545	32
29	844	99996	2589	99966	4333	99906	6076	99815	7817	99694	9556	99542	31
30	873	99996	2618	99966	4362	99905	6105	99813	7846	99692	9585	99540	30
31	902	99996	2647	99965	4391	99904	6134	99812	7875	99689	9614	99537	29
32	931	99996	2676	99964	4420	99902	6163	99810	7904	99687	9642	99534	28
33	960	99995	2705	99963	4449	99901	6192	99808	7933	99685	9671	99531	27
34	989	99995	2734	99963	4478	99900	6221	99806	7962	99683	9700	99528	26
35	1018	99995	2763	99962	4507	99898	6250	99804	7991	29680	9729	99526	25
36	1047	99995	2792	99961	4536	99897	6279	99803	8020	99678	9758	99523	24
37	1076	99994	2821	99960	4565	99896	6308	99801	8049	99676	9787	99520	23
38	1105	99994	2850	99959	4594	99894	6337	99799	8078	99673	9816	99517	22
39	1134	99994	2879	99959	4623	99893	6366	99797	8107	99671	9845	99514	21
40	1164	99993	2908	99958	4653	99892	6395	99795	8136	99668	9874	99511	20
41	1193	99993	2938	99957	4682	99890	6424	99793	8165	99666	9903	99508	19
42	1222	99993	2967	99956	4711	99889	6453	99792	8194	99664	9932	99506	18
43	1251	99992	2996	99955	4740	99888	6482	99790	8223	99661	9961	99503	17
44	1280	99992	3025	99954	4769	99886	6511	99788	8252	99659	9990	99500	16
45	1309	99991	3054	99953	4798	99885	6540	99786	8281	99657	10019	99497	15
46	1338	99991	3083	99952	4827	99883	6569	99784	8310	99654	10048	99494	14
47	1367	99991	3112	99952	4856	99882	6598	99782	8339	99652	10077	99491	13
48	1396	99990	3141	99951	4885	99881	6627	99780	8368	99649	10106	99488	12
49	1425	99990	3170	99950	4914	99879	6656	99778	8397	99647	10135	99485	11
50	1454	99989	3199	99949	4943	99878	6685	99776	8426	99644	10164	99482	10
51	1483	99989	3228	99948	4972	99876	6714	99774	8455	99642	10192	99479	9
52	1513	99989	3257	99947	5001	99875	6743	99772	8484	99639	10221	99476	8
53	1542	99988	3286	99946	5030	99873	6773	99770	8513	99637	10250	99473	7
54	1571	99988	3316	99945	5059	99872	6802	99768	8542	99635	10279	99470	6
55	1600	99987	3345	99944	5088	99870	6831	99766	8571	99632	10308	99467	5
56	1629	99987	3374	99943	5117	99869	6860	99764	8600	99630	10337	99464	4
57	1658	99986	3403	99942	5146	99867	6889	99762	8629	99627	10366	99461	3
58	1687	99986	3432	99941	5175	99866	6918	99760	8658	99625	10395	99458	2
59	1716	99985	3461	99940	5205	99864	6947	99758	8687	99622	10424	99455	1
60	1745	99985	3490	99939	5234	99863	6976	99756	8716	99619	10453	99452	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	89 Deg.		88 Deg.		87 Deg.		86 Deg.		85 Deg.		84 Deg.		

M.	6 Deg. N. sin.	6 Deg. N. cos.	7 Deg. N. sin.	7 Deg. N. cos.	8 Deg. N. sin.	8 Deg. N. cos.	9 Deg. N. sin.	9 Deg. N. cos.	10 Deg. N. sin.	10 Deg. N. cos.	11 Deg. N. sin.	11 Deg. N. cos.	
0	10453	99452	12187	99255	13917	99027	15643	98769	17365	98481	19081	98163	60
1	10482	99449	12216	99251	13946	99023	15672	98764	17393	98476	19109	98157	59
2	10511	99446	12245	99248	13975	99019	15701	98760	17422	98471	19138	98152	58
3	10540	99443	12274	99244	14004	99015	15730	98755	17451	98466	19167	98146	57
4	10569	99440	12302	99240	14033	99011	15758	98751	17479	98461	19195	98140	56
5	10597	99437	12331	99237	14061	99006	15787	98746	17508	98455	19224	98135	55
6	10626	99434	12360	99233	14090	99002	15816	98741	17537	98450	19252	98129	54
7	10655	99431	12389	99230	14119	98998	15845	98737	17565	98445	19281	98124	53
8	10684	99428	12418	99226	14148	98994	15873	98732	17594	98440	19309	98118	52
9	10713	99424	12447	99222	14177	98990	15902	98728	17623	98435	19338	98112	51
10	10742	99421	12476	99219	14205	98986	15931	98723	17651	98430	19366	98107	50
11	10771	99418	12504	99215	14234	98982	15959	98718	17680	98425	19395	98101	49
12	10800	99415	12533	99211	14263	98978	15988	98714	17708	98420	19423	98096	48
13	10829	99412	12562	99208	14292	98973	16017	98709	17737	98414	19452	98090	47
14	10858	99409	12591	99204	14320	98969	16046	98704	17766	98409	19481	98084	46
15	10887	99406	12620	99200	14349	98965	16074	98700	17794	98404	19509	98079	45
16	10916	99402	12649	99197	14378	98961	16103	98695	17823	98399	19538	98073	44
17	10945	99399	12678	99193	14407	98957	16132	98690	17852	98394	19566	98067	43
18	10973	99396	12706	99189	14436	98953	16160	98686	17880	98389	19595	98061	42
19	11002	99393	12735	99186	14464	98948	16189	98681	17909	98383	19623	98056	41
20	11031	99390	12764	99182	14493	98944	16218	98676	17937	98378	19652	98050	40
21	11060	99386	12793	99178	14522	98940	16246	98671	17966	98373	19680	98044	39
22	11089	99383	12822	99175	14551	98936	16275	98667	17995	98368	19709	98039	38
23	11118	99380	12851	99171	14580	98931	16304	98662	18023	98362	19737	98033	37
24	11147	99377	12880	99167	14608	98927	16333	98657	18052	98357	19766	98027	36
25	11176	99374	12908	99163	14637	98923	16361	98652	18081	98352	19794	98021	35
26	11205	99370	12937	99160	14666	98919	16390	98648	18109	98347	19823	98016	34
27	11234	99367	12966	99156	14695	98914	16419	98643	18133	98341	19851	98010	33
28	11263	99364	12995	99152	14723	98910	16447	98638	18166	98336	19880	98004	32
29	11291	99360	13024	99148	14752	98906	16476	98633	18195	98331	19908	97998	31
30	11320	99357	13053	99144	14781	98902	16505	98629	18224	98325	19937	97992	30
31	11349	99354	13081	99141	14810	98897	16533	98624	18252	98320	19965	97987	29
32	11378	99351	13110	99137	14838	98893	16562	98619	18281	98315	19994	97981	28
33	11407	99347	13139	99133	14867	98889	16591	98614	18309	98310	20022	97975	27
34	11436	99344	13168	99129	14896	98884	16620	98609	18338	98304	20051	97969	26
35	11465	99341	13197	99125	14925	98880	16648	98604	18367	98299	20079	97963	25
36	11494	99337	13226	99122	14954	98876	16677	98600	18395	98294	20108	97958	24
37	11523	99334	13254	99118	14982	98871	16706	98595	18424	98288	20136	97952	23
38	11552	99331	13283	99114	15011	98867	16734	98590	18452	98283	20165	97946	22
39	11580	99327	13312	99110	15040	98863	16763	98585	18481	98277	20193	97940	21
40	11609	99324	13341	99106	15069	98858	16792	98580	18509	98272	20222	97934	20
41	11638	99320	13370	99102	15097	98854	16820	98575	18538	98267	20250	97928	19
42	11667	99317	13399	99098	15126	98849	16849	98570	18567	98261	20279	97922	18
43	11696	99314	13427	99094	15155	98845	16878	98565	18595	98256	20307	97916	17
44	11725	99310	13456	99091	15184	98841	16906	98561	18624	98250	20336	97910	16
45	11754	99307	13485	99087	15212	98836	16935	98556	18652	98245	20364	97905	15
46	11783	99303	13514	99083	15241	98832	16964	98551	18681	98240	20393	97899	14
47	11812	99300	13543	99079	15270	98827	16992	98546	18710	98234	20421	97893	13
48	11840	99297	13572	99075	15299	98823	17021	98541	18738	98229	20450	97887	12
49	11869	99293	13600	99071	15327	98818	17050	98536	18767	98223	20478	97881	11
50	11898	99290	13629	99067	15356	98814	17078	98531	18795	98218	20507	97875	10
51	11927	99286	13658	99063	15385	98809	17107	98526	18824	98212	20535	97869	9
52	11956	99283	13687	99059	15414	98805	17136	98521	18852	98207	20563	97863	8
53	11985	99279	13716	99055	15442	98800	17164	98516	18881	98201	20592	97857	7
54	12014	99276	13744	99051	15471	98796	17193	98511	18910	98196	20620	97851	6
55	12043	99272	13773	99047	15500	98791	17222	98506	18938	98190	20649	97845	5
56	12071	99269	13802	99043	15529	98787	17250	98501	18967	98185	20677	97839	4
57	12100	99265	13831	99039	15557	98782	17279	98496	18995	98179	20706	97833	3
58	12129	99262	13860	99035	15586	98778	17308	98491	19024	98174	20734	97827	2
59	12158	99258	13889	99031	15615	98773	17336	98486	19052	98168	20763	97821	1
60	12187	99255	13917	99027	15643	98769	17365	98481	19081	98163	20791	97815	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	83 Deg.		82 Deg.		81 Deg.		80 Deg.		79 Deg.		78 Deg.		

M.	12 Deg.		13 Deg.		14 Deg.		15 Deg.		16 Deg.		17 Deg.		
	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	
0	20791	97815	22495	97437	24192	97030	25882	96593	27564	96126	29237	95630	60
1	20820	97809	22523	97430	24220	97023	25910	96585	27592	96118	29265	95622	59
2	20848	97803	22552	97424	24249	97015	25938	96578	27620	96110	29293	95613	58
3	20877	97797	22580	97417	24277	97008	25966	96570	27648	96102	29321	95605	57
4	20905	97791	22608	97411	24305	97001	25994	96562	27676	96094	29348	95596	56
5	20933	97784	22637	97404	24333	96994	26022	96555	27704	96086	29376	95588	55
6	20962	97778	22665	97398	24362	96987	26050	96547	27731	96078	29404	95579	54
7	20990	97772	22693	97391	24390	96980	26079	96540	27759	96070	29432	95571	53
8	21019	97766	22722	97384	24418	96973	26107	96532	27787	96062	29460	95562	52
9	21047	97760	22750	97378	24446	96966	26135	96524	27815	96054	29487	95554	51
10	21076	97754	22778	97371	24474	96959	26163	96517	27843	96046	29515	95545	50
11	21104	97748	22807	97365	24503	96952	26191	96509	27871	96037	29543	95536	49
12	21132	97742	22835	97358	24531	96945	26219	96502	27899	96029	29571	95528	48
13	21161	97735	22863	97351	24559	96937	26247	96494	27927	96021	29599	95519	47
14	21189	97729	22892	97345	24587	96930	26275	96486	27955	96013	29626	95511	46
15	21218	97723	22920	97338	24615	96923	26303	96479	27983	96005	29654	95502	45
16	21246	97717	22948	97331	24644	96916	26331	96471	28011	95997	29682	95493	44
17	21275	97711	22977	97325	24672	96909	26359	96463	28039	95989	29710	95485	43
18	21303	97705	23005	97318	24700	96902	26387	96456	28067	95981	29737	95476	42
19	21331	97698	23033	97311	24728	96894	26415	96448	28095	95972	29765	95467	41
20	21360	97692	23062	97304	24756	96887	26443	96440	28123	95964	29793	95459	40
21	21388	97686	23090	97298	24784	96880	26471	96433	28150	95956	29821	95450	39
22	21417	97680	23118	97291	24813	96873	26500	96425	28178	95948	29849	95441	38
23	21445	97673	23146	97284	24841	96866	26528	96417	28206	95940	29876	95433	37
24	21474	97667	23175	97278	24869	96858	26556	96410	28234	95931	29904	95424	36
25	21502	97661	23203	97271	24897	96851	26584	96402	28262	95923	29932	95415	35
26	21530	97655	23231	97264	24925	96844	26612	96394	28290	95915	29960	95407	34
27	21559	97648	23260	97257	24954	96837	26640	96386	28318	95907	29987	95398	33
28	21587	97642	23288	97251	24982	96829	26668	96379	28346	95898	30015	95389	32
29	21616	97636	23316	97244	25010	96822	26696	96371	28374	95890	30043	95380	31
30	21644	97630	23345	97237	25038	96815	26724	96363	28402	95882	30071	95372	30
31	21672	97623	23373	97230	25066	96807	26752	96355	28429	95874	30098	95363	29
32	21701	97617	23401	97223	25094	96800	26780	96347	28457	95865	30126	95354	28
33	21729	97611	23429	97217	25122	96793	26808	96340	28485	95857	30154	95345	27
34	21758	97604	23458	97210	25151	96786	26836	96332	28513	95849	30182	95337	26
35	21786	97598	23486	97203	25179	96778	26864	96324	28541	95841	30209	95328	25
36	21814	97592	23514	97196	25207	96771	26892	96316	28569	95832	30237	95319	24
37	21843	97585	23542	97189	25235	96764	26920	96308	28597	95824	30265	95310	23
38	21871	97579	23571	97182	25263	96756	26948	96301	28625	95816	30292	95301	22
39	21899	97573	23599	97176	25291	96749	26976	96293	28652	95807	30320	95293	21
40	21928	97566	23627	97169	25320	96742	27004	96285	28680	95799	30348	95284	20
41	21956	97560	23656	97162	25348	96734	27032	96277	28708	95791	30376	95275	19
42	21985	97553	23684	97155	25376	96727	27060	96269	28736	95782	30403	95266	18
43	22013	97547	23712	97148	25404	96719	27088	96261	28764	95774	30431	95257	17
44	22041	97541	23740	97141	25432	96712	27116	96253	28792	95766	30459	95248	16
45	22070	97534	23769	97134	25460	96705	27144	96246	28820	95757	30486	95240	15
46	22098	97528	23797	97127	25488	96697	27172	96238	28847	95749	30514	95231	14
47	22126	97521	23825	97120	25516	96690	27200	96230	28875	95740	30542	95222	13
48	22155	97515	23853	97113	25545	96682	27228	96222	28903	95732	30570	95213	12
49	22183	97508	23882	97106	25573	96675	27256	96214	28931	95724	30597	95204	11
50	22212	97502	23910	97100	25601	96667	27284	96206	28959	95715	30625	95195	10
51	22240	97496	23938	97093	25629	96660	27312	96198	28987	95707	30653	95186	9
52	22268	97489	23966	97086	25657	96653	27340	96190	29015	95698	30680	95177	8
53	22297	97483	23995	97079	25685	96645	27368	96182	29042	95690	30708	95168	7
54	22325	97476	24023	97072	25713	96638	27396	96174	29070	95681	30736	95159	6
55	22353	97470	24051	97065	25741	96630	27424	96166	29098	95673	30763	95150	5
56	22382	97463	24079	97058	25769	96623	27452	96158	29126	95664	30791	95142	4
57	22410	97457	24108	97051	25798	96615	27480	96150	29154	95656	30819	95133	3
58	22438	97450	24136	97044	25826	96608	27508	96142	29182	95647	30846	95124	2
59	22467	97444	24164	97037	25854	96600	27536	96134	29209	95639	30874	95115	1
60	22495	97437	24192	97030	25882	96593	27564	96126	29237	95630	30902	95106	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	77 Deg.		76 Deg.		75 Deg.		74 Deg.		73 Deg.		72 Deg.		

M.	18 Deg. N. sin.	18 Deg. N. cos.	19 Deg. N. sin.	19 Deg. N. cos.	20 Deg. N. sin.	20 Deg. N. cos.	21 Deg. N. sin.	21 Deg. N. cos.	22 Deg. N. sin.	22 Deg. N. cos.	23 Deg. N. sin.	23 Deg. N. cos.	
0	30902	95106	32557	94552	34202	93969	35837	93358	37461	92718	39073	92050	60
1	30929	95097	32584	94542	34229	93959	35864	93348	37488	92707	39100	92039	59
2	30957	95088	32612	94533	34257	93949	35891	93337	37515	92697	39127	92028	58
3	30985	95079	32639	94523	34284	93939	35918	93327	37542	92686	39153	92016	57
4	31012	95070	32667	94514	34311	93929	35945	93316	37569	92675	39180	92005	56
5	31040	95061	32694	94504	34339	93919	35973	93306	37595	92664	39207	91994	55
6	31068	95052	32722	94495	34366	93909	36000	93295	37622	92653	39234	91982	54
7	31095	95043	32749	94485	34393	93899	36027	92285	37649	92642	39260	91971	53
8	31123	95033	32777	94476	34421	93889	36054	93274	37676	92631	39287	91959	52
9	31151	95024	32804	94466	34448	93879	36081	93264	37703	92620	39314	91948	51
10	31178	95015	32832	94457	34475	93869	36108	93253	37730	92609	39341	91936	50
11	31206	95006	32859	94447	34503	93859	36135	93243	37757	92598	39367	91925	49
12	31233	94997	32887	94438	34530	93849	36162	93232	37784	92587	39394	91914	48
13	31261	94988	32914	94428	34557	93839	36190	93222	37811	92576	39421	91902	47
14	31289	94979	32942	94418	34584	93829	36217	93211	37838	92565	39448	91891	46
15	31316	94970	32969	94409	34612	93819	36244	93201	37865	92554	39474	91879	45
16	31344	94961	32997	94399	34639	93809	36271	93190	37892	92543	39501	91868	44
17	31372	94952	33024	94390	34666	93799	36298	93180	37919	92532	39528	91856	43
18	31399	94943	33051	94380	34694	93789	36325	93169	37946	92521	39555	91845	42
19	31427	94933	33079	94370	34721	93779	36352	93159	37973	92510	39581	91833	41
20	31454	94924	33106	94361	34748	93769	36379	93148	37999	92499	39608	91822	40
21	31482	94915	33134	94351	34775	93759	36406	93137	38026	92488	39635	91810	39
22	31510	94906	33161	94342	34803	93748	36434	93127	38053	92477	39661	91799	38
23	31537	94897	33189	94332	34830	93738	36461	93116	38080	92466	39688	91787	37
24	31565	94888	33216	94322	34857	93728	36488	93106	38107	92455	39715	91775	36
25	31593	94878	33244	94313	34884	93718	36515	93095	38134	92444	39741	91764	35
26	31620	94869	33271	94303	34912	93708	36542	93084	38161	92432	39768	91752	34
27	31648	94860	33298	94293	34939	93698	36569	93074	38188	92421	39795	91741	33
28	31675	94851	33326	94284	34966	93688	36596	93063	38215	92410	39822	91729	32
29	31703	94842	33353	94274	34993	93677	36623	93052	38241	92399	39848	91718	31
30	31730	94832	33381	94264	35021	93667	36650	93042	38268	92388	39875	91706	30
31	31758	94823	33408	94254	35048	93657	36677	93031	38295	92377	39902	91694	29
32	31786	94814	33436	94245	35075	93647	36704	93020	38322	92366	39928	91683	28
33	31813	94805	33463	94235	35102	93637	36731	93010	38349	92355	39955	91671	27
34	31841	94795	33490	94225	35130	93626	36758	92999	38376	92343	39982	91660	26
35	31868	94786	33518	94215	35157	93616	36785	92988	38403	92332	40008	91648	25
36	31896	94777	33545	94206	35184	93606	36812	92978	38430	92321	40035	91636	24
37	31923	94768	33573	94196	35211	93596	36839	92967	38456	92310	40062	91625	23
38	31951	94758	33600	94186	35239	93585	36867	92956	38483	92299	40088	91613	22
39	31979	94749	33627	94176	35266	93575	36894	92945	38510	92287	40115	91601	21
40	32006	94740	33655	94167	35293	93565	36921	92935	38537	92276	40141	91590	20
41	32034	94730	33682	94157	35320	93555	36948	92924	38564	92265	40168	91578	19
42	32061	94721	33710	94147	35347	93544	36975	92913	38591	92254	40195	91566	18
43	32089	94712	33737	94137	35375	93534	37002	92902	38617	92243	40221	91555	17
44	32116	94702	33764	94127	35402	93524	37029	92892	38644	92231	40248	91543	16
45	32144	94693	33792	94118	35429	93514	37056	92881	38671	92220	40275	91531	15
46	32171	94684	33819	94108	35456	93503	37083	92870	38698	92209	40301	91519	14
47	32199	94674	33846	94098	35484	93493	37110	92859	38725	92198	40328	91508	13
48	32227	94665	33874	94088	35511	93483	37137	92849	38752	92186	40355	91496	12
49	32254	94656	33901	94078	35538	93472	37164	92838	38778	92175	40381	91484	11
50	32282	94646	33929	94068	35565	93462	37191	92827	38805	92164	40408	91472	10
51	32309	94637	33956	94058	35592	93452	37218	92816	38832	92152	40434	91461	9
52	32337	94627	33983	94049	35619	93441	37245	92805	38859	92141	40461	91449	8
53	32364	94618	34011	94039	35647	93431	37272	92794	38886	92130	40488	91437	7
54	32392	94609	34038	94029	35674	93420	37299	92784	38912	92119	40514	91425	6
55	32419	94599	34065	94019	35701	93410	37326	92773	38939	92107	40541	91414	5
56	32447	94590	34093	94009	35728	93400	37353	92762	38966	92096	40567	91402	4
57	32474	94580	34120	93999	35755	93389	37380	92751	38993	92085	40594	91390	3
58	32502	94571	34147	93989	35782	93379	37407	92740	39020	92073	40621	91378	2
59	32529	94561	34175	93979	35810	93368	37434	92729	39046	92062	40647	91366	1
60	32557	94552	34202	93969	35837	93358	37461	92718	39073	92050	40674	91355	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	71 Deg.		70 Deg.		69 Deg.		68 Deg.		67 Deg.		66 Deg.		

	24 Deg.		25 Deg.		26 Deg.		27 Deg.		28 Deg.		29 Deg.		
M.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	
0	40674	91355	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60
1	40700	91343	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59
2	40727	91331	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58
3	40753	91319	42341	90594	43916	89841	45477	89061	47024	88254	48557	87420	57
4	40780	91307	42367	90582	43942	89828	45503	89048	47050	88240	48583	87406	56
5	40806	91295	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55
6	40833	91283	42420	90557	43994	89803	45554	89021	47101	88213	48634	87377	54
7	40860	91272	42446	90545	44020	89790	45580	89008	47127	88199	48659	87363	53
8	40886	91260	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52
9	40913	91248	42499	90520	44072	89764	45632	88981	47178	88172	48710	87335	51
10	40939	91236	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50
11	40966	91224	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49
12	40992	91212	42578	90483	44151	89726	45710	88942	47255	88130	48786	87292	48
13	41019	91200	42604	90470	44177	89713	45736	88928	47281	88117	48811	87278	47
14	41045	91188	42631	90458	44203	89700	45762	88915	47306	88103	48837	87264	46
15	41072	91176	42657	90446	44229	89687	45787	88902	47332	88089	48862	87250	45
16	41098	91164	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44
17	41125	91152	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43
18	41151	91140	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42
19	41178	91128	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41
20	41204	91116	42788	90383	44359	89623	45917	88835	47460	88020	48989	87178	40
21	41231	91104	42815	90371	44385	89610	45942	88822	47486	88006	49014	87164	39
22	41257	91092	42841	90358	44411	89597	45968	88808	47511	87993	49040	87150	38
23	41284	91080	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37
24	41310	91068	42894	90334	44464	89571	46020	88782	47562	87965	49090	87121	36
25	41337	91056	42920	90321	44490	89558	46046	88768	47588	87951	49116	87107	35
26	41363	91044	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34
27	41390	91032	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33
28	41416	91020	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32
29	41443	91008	43025	90271	44594	89506	46149	88715	47690	87896	49217	87050	31
30	41469	90996	43051	90259	44620	89493	46175	88701	47716	87882	49242	87036	30
31	41496	90984	43077	90246	44646	89480	46201	88688	47741	87868	49268	87021	29
32	41522	90972	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28
33	41549	90960	43130	90221	44698	89454	46252	88661	47793	87840	49318	86993	27
34	41575	90948	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26
35	41602	90936	43182	90196	44750	89428	46304	88634	47844	87812	49369	86964	25
36	41628	90924	43209	90183	44776	89415	46330	88620	47869	87798	49394	86949	24
37	41655	90911	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23
38	41681	90899	43261	90158	44828	89389	46381	88593	47920	87770	49445	86921	22
39	41707	90887	43287	90146	44854	89376	46407	88580	47946	87756	49470	86906	21
40	41734	90875	43313	90133	44880	89363	46433	88566	47971	87743	49495	86892	20
41	41760	90863	43340	90120	44906	89350	46458	88553	47997	87729	49521	86878	19
42	41787	90851	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18
43	41813	90839	43392	90095	44958	89324	46510	88526	48048	87701	49571	86849	17
44	41840	90826	43418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16
45	41866	90814	43445	90070	45010	89298	46561	88499	48099	87673	49622	86820	15
46	41892	90802	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14
47	41919	90790	43497	90045	45062	89272	46613	88472	48150	87645	49672	86791	13
48	41945	90778	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12
49	41972	90766	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11
50	41998	90753	43575	90007	45140	89232	46690	88431	48226	87603	49748	86748	10
51	42024	90741	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9
52	42051	90729	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8
53	42077	90717	43654	89968	45218	89193	46767	88390	48303	87561	49824	86704	7
54	42104	90704	43680	89956	45243	89180	46793	88377	48328	87546	49849	86690	6
55	42130	90692	43706	89943	45269	89167	46819	88363	48354	87532	49874	86675	5
56	42156	90680	43733	89930	45295	89153	46844	88349	48379	87518	49899	86661	4
57	42183	90668	43759	89918	45321	89140	46870	88336	48405	87504	49924	86646	3
58	42209	90655	43785	89905	45347	89127	46896	88322	48430	87490	49950	86632	2
59	42235	90643	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1
60	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	50000	86603	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	65 Deg.		64 Deg.		63 Deg.		62 Deg.		61 Deg.		60 Deg.		

M.	30 Deg. N. sin.	30 Deg. N. cos.	31 Deg. N. sin.	31 Deg. N. cos.	32 Deg. N. sin.	32 Deg. N. cos.	33 Deg. N. sin.	33 Deg. N. cos.	34 Deg. N. sin.	34 Deg. N. cos.	35 Deg. N. sin.	35 Deg. N. cos.	
0	50000	86603	51504	85717	52992	84805	54464	83867	55919	82904	57358	81915	60
1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	57381	81899	59
2	50050	86573	51554	85687	53041	84774	54513	83835	55968	82871	57405	81882	58
3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57429	81865	57
4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	57453	81848	56
5	50126	86530	51628	85642	53115	84728	54586	83788	56040	82822	57477	81832	55
6	50151	86515	51653	85627	53140	84712	54610	83772	56064	82806	57501	81815	54
7	50176	86501	51678	85612	53164	84697	54635	83756	56088	82790	57524	81798	53
8	50201	86486	51703	85597	53189	84681	54659	85740	56112	82773	57548	81782	52
9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	57572	81765	51
10	50252	86457	51753	85567	53238	84650	54708	83708	56160	82741	57596	81748	50
11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	57619	81731	49
12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	57643	81714	48
13	50327	86413	51828	85521	53312	84604	54781	83660	56232	82692	57667	81698	47
14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	57691	81681	46
15	50377	86384	51877	85491	53361	84573	54829	83629	56280	82659	57715	81664	45
16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	57738	81647	44
17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	57762	81631	43
18	50453	86340	51952	85446	53435	84526	54902	83581	56353	82610	57786	81614	42
19	50478	86325	51977	85431	53460	84511	54927	83565	56377	82593	57810	81597	41
20	50503	86310	52002	85416	53484	84495	54951	83549	56401	82577	57833	81580	40
21	50528	86295	52026	85401	53509	84480	54975	83533	56425	82561	57857	81563	39
22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	57881	81546	38
23	50578	86266	52076	85370	53558	84448	55024	83501	56473	82528	57904	81530	37
24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	57928	81513	36
25	50628	86237	52126	85340	53607	84417	55072	83469	56521	82495	57952	81496	35
26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	57976	81479	34
27	50679	86207	52175	85310	53656	84386	55121	83437	56569	82462	57999	81462	33
28	50704	86192	52200	85294	53681	84370	55145	83421	56593	82446	58023	81445	32
29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	58047	81428	31
30	50754	86163	52250	85264	53730	84339	55194	83389	56641	82413	58070	81412	30
31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	58094	81395	29
32	50804	86133	52299	85234	53779	84308	55242	83356	56689	82380	58118	81378	28
33	50829	86119	52324	85218	53804	84292	55266	83340	56713	82363	58141	81361	27
34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	58165	81344	26
35	50879	86089	52374	85188	53853	84261	55315	83308	56760	82330	58189	81327	25
36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	58212	81310	24
37	50929	86059	52423	85157	53902	84230	55363	83276	56808	82297	58236	81293	23
38	50954	86045	52448	85142	53926	84214	55388	83260	56832	82281	58260	81276	22
39	50979	86030	52473	85127	53951	84198	55412	83244	56856	82264	58283	81259	21
40	51004	86015	52498	85112	53975	84182	55436	83228	56880	82248	58307	81242	20
41	51029	86000	52522	85096	54000	84167	55460	83212	56904	82231	58330	81225	19
42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	58354	81208	18
43	51079	85970	52572	85066	54049	84135	55509	83179	56952	82198	58378	81191	17
44	51104	85956	52597	85051	54073	84120	55533	83163	56976	82181	58401	81174	16
45	51129	85941	52621	85035	54097	84104	55557	83147	57000	82165	58425	81157	15
46	51154	85926	52646	85020	54122	84088	55581	83131	57024	82148	58449	81140	14
47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	58472	81123	13
48	51204	85896	52696	84989	54171	84057	55630	83098	57071	82115	58496	81106	12
49	51229	85881	52720	84974	54195	84041	55654	83082	57095	82098	58519	81089	11
50	51254	85866	52745	84959	54220	84025	55678	83066	57119	82082	58543	81072	10
51	51279	85851	52770	84943	54244	84009	55702	83050	57143	82065	58567	81055	9
52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	58590	81038	8
53	51329	85821	52819	84913	54293	83978	55750	83017	57191	82032	58614	81021	7
54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	58637	81004	6
55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	58661	80987	5
56	51404	85777	52893	84866	54366	83930	55823	82969	57262	81982	58684	80970	4
57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	58708	80953	3
58	51454	85747	52943	64836	54415	83899	55871	82936	57310	81949	58731	80936	2
59	51479	85732	52967	84820	54440	83883	55895	82920	57334	81932	58755	80919	1
60	51504	85717	52992	84805	54464	83867	55919	82904	57358	81915	58779	80902	0
	N. cos.	N. sin.	N. cos.	N. siu.	N. cos.	N. sin	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	59 Deg.		58 Deg.		57 Deg.		56 Deg.		55 Deg.		54 Deg.		

M.	36 Deg. N. sin.	N. cos.	37 Deg. N. sin.	N. cos.	38 Deg. N. sin.	N. cos.	39 Deg. N. sin.	N. cos.	40 Deg. N. sin.	N. cos.	41 Deg. N. sin.	N. cos.	
0	58779	80902	60182	79864	61566	78801	62932	77715	64279	76604	65606	75471	60
1	58802	80885	60205	79846	61589	78783	62955	77696	64301	76586	65628	75452	59
2	58826	80867	60228	79829	61612	78765	62977	77678	64323	76567	65650	75433	58
3	58849	80850	60251	79811	61635	78747	63000	77660	64346	76548	65672	75414	57
4	58873	80833	60274	79793	61658	78729	63022	77641	64368	76530	65694	75395	56
5	58896	80816	60298	79776	61681	78711	63045	77623	64390	76511	65716	75375	55
6	58920	80799	60321	79758	61704	78693	63068	77605	64412	76492	65738	75356	54
7	58943	80782	60344	79741	61726	78676	63090	77586	64435	76473	65759	75337	53
8	58967	80765	60367	79723	61749	78658	63113	77568	64457	76455	65781	75318	52
9	58990	80748	60390	79706	61772	78640	63135	77550	64479	76436	65803	75299	51
10	59014	80730	60414	79688	61795	78622	63158	77531	64501	76417	65825	75280	50
11	59037	80713	60437	79671	61818	78604	63180	77513	64524	76398	65847	75261	49
12	59061	80696	60460	79653	61841	78586	63203	77494	64546	76380	65869	75241	48
13	59084	80679	60483	79635	61864	78568	63225	77476	64568	76361	65891	75222	47
14	59108	80662	60506	79618	61887	78550	63248	77458	64590	76342	65913	75203	46
15	59131	80644	60529	79600	61909	78532	63271	77439	64612	76323	65935	75184	45
16	59154	80627	60553	79583	61932	78514	63293	77421	64635	76304	65956	75165	44
17	59178	80610	60576	79565	61955	78496	63316	77402	64657	76286	65978	75146	43
18	59201	80593	60599	79547	61978	78478	63338	77384	64679	76267	66000	75126	42
19	59225	80576	60622	79530	62001	78460	63361	77366	64701	76248	66022	75107	41
20	59248	80558	60645	79512	62024	78442	63383	77347	64723	76229	66044	75088	40
21	59272	80541	60668	79494	62046	78424	63406	77329	64746	76210	66066	75069	39
22	59295	80524	60691	79477	62069	78405	63428	77310	64768	76192	66088	75050	38
23	59318	80507	60714	79459	62092	78387	63451	77292	64790	76173	66109	75030	37
24	59342	80489	60738	79441	62115	78369	63473	77273	64812	76154	66131	75011	36
25	59365	80472	60761	79424	62138	78351	63496	77255	64834	76135	66153	74992	35
26	59389	80455	60784	79406	62160	78333	63518	77236	64856	76116	66175	74973	34
27	59412	80438	60807	79388	62183	78315	63540	77218	64878	76097	66197	74953	33
28	59436	80420	60830	79371	62206	78297	63563	77199	64901	76078	66218	74934	32
29	59459	80403	60853	79353	62229	78279	63585	77181	64923	76059	66240	74915	31
30	59482	80386	60876	79335	62251	78261	63608	77162	64945	76041	66262	74896	30
31	59506	80368	60899	79318	62274	78243	63630	77144	64967	76022	66284	74876	29
32	59529	80351	60922	79300	62297	78225	63653	77125	64989	76003	66306	74857	28
33	59552	80334	60945	79282	62320	78206	63675	77107	65011	75984	66327	74838	27
34	59576	80316	60968	79264	62342	78188	63698	77088	65033	75965	66349	74818	26
35	59599	80299	60991	79247	62365	78170	63720	77070	65055	75946	66371	74799	25
36	59622	80282	61015	79229	62388	78152	63742	77051	65077	75927	66393	74780	24
37	59646	80264	61038	79211	62411	78134	63765	77033	65100	75908	66414	74760	23
38	59669	80247	61061	79193	62433	78116	63787	77014	65122	75889	66436	74741	22
39	59693	80230	61084	79176	62456	78098	63810	76996	65144	75870	66458	74722	21
40	59716	80212	61107	79158	62479	78079	63832	76977	65166	75851	66480	74703	20
41	59739	80195	61130	79140	62502	78061	63854	76959	65188	75832	66501	74683	19
42	59762	80178	61153	79122	62524	78043	63877	76940	65210	75813	66523	74664	18
43	59786	80160	61176	79105	62547	78025	63899	76921	65232	75794	66545	74644	17
44	59809	80143	61199	79087	62570	78007	63922	76903	65254	75775	66566	74625	16
45	59832	80125	61222	79069	62592	77988	63944	76884	65276	75756	66588	74606	15
46	59856	80108	61245	79051	62615	77970	63966	76866	65298	75738	66610	74586	14
47	59879	80091	61268	79033	62638	77952	63989	76847	65320	75719	66632	74567	13
48	59902	80073	61291	79016	62660	77934	64011	76828	65342	75700	66653	74548	12
49	59926	80056	61314	78998	62683	77916	64033	76810	65364	75680	66675	74528	11
50	59949	80038	61337	78980	62706	77897	64056	76791	65386	75661	66697	74509	10
51	59972	80021	61360	78962	62728	77879	64078	76772	65408	75642	66718	74489	9
52	59995	80003	61383	78944	62751	77861	64100	76754	65430	75623	66740	74470	8
53	60019	79986	61406	78926	62774	77843	64123	76735	65452	75604	66762	74451	7
54	60042	79968	61429	78908	62796	77824	64145	76717	65474	75585	66783	74431	6
55	60065	79951	61451	78891	62819	77806	64167	76698	65496	75566	66805	74412	5
56	60089	79934	61474	78873	62842	77788	64190	76679	65518	75547	66827	74392	4
57	60112	79916	61497	78855	62864	77769	64212	76661	65540	75528	66848	74373	3
58	60135	79899	61520	78837	62887	77751	64234	76642	65562	75509	66870	74353	2
59	60158	79881	61543	78819	62909	77733	64256	76623	65584	75490	66891	74334	1
60	60182	79864	61566	78801	62932	77715	64279	76604	65606	75471	66913	74314	0
	N. cos.	N. sin. 53 Deg.	N. cos.	N. sin. 52 Deg.	N. cos.	N. sin. 51 Deg.	N. cos.	N. sin. 50 Deg.	N. cos.	N. sin. 49 Deg.	N. cos.	N. sin. 48 Deg.	M.

M.	42 Deg. N. sin.	42 Deg. N. cos.	43 Deg. N. sin.	43 Deg. N. cos.	44 Deg. N. sin.	44 Deg. N. cos.		M.	42 Deg. N. sin.	42 Deg. N. cos.	43 Deg. N. sin.	43 Deg. N. cos.	44 Deg. N. sin.	44 Deg. N. cos.	
0	66913	74314	68200	73135	69466	71934	60	30	67559	73728	68835	72537	70091	71325	30
1	66935	74295	68221	73116	69487	71914	59	31	67580	73708	68857	72517	70112	71305	29
2	66956	74276	68242	73096	69508	71894	58	32	67602	73688	68878	72497	70132	71284	28
3	66978	74256	68264	73076	69529	71873	57	33	67623	73669	68899	72477	70153	71264	27
4	66999	74237	68285	73056	69549	71853	56	34	67645	73649	68920	72457	70174	71243	26
5	67021	74217	68306	73036	69570	71833	55	35	67666	73629	68941	72437	70195	71223	25
6	67043	74198	68327	73016	69591	71813	54	36	67688	73610	68962	72417	70215	71203	24
7	67064	74178	68349	72996	69612	71792	53	37	67709	73590	68983	72397	70236	71182	23
8	67086	74159	68370	72976	69633	71772	52	38	67730	73570	69004	72377	70257	71162	22
9	67107	74139	68391	72957	69654	71752	51	39	67752	73551	69025	72357	70277	71141	21
10	67129	74120	68412	72937	69675	71732	50	40	67773	73531	69046	72337	70298	71121	20
11	67151	74100	68434	72917	69696	71711	49	41	67795	73511	69067	72317	70319	71100	19
12	67172	74080	68455	72897	69717	71691	48	42	67816	73491	69088	72297	70339	71080	18
13	67194	74061	68476	72877	69737	71671	47	43	67837	73472	69109	72277	70360	71059	17
14	67215	74041	68497	72857	69758	71650	46	44	67859	73452	69130	72257	70381	71039	16
15	67237	74022	68518	72837	69779	71630	45	45	67880	73432	69151	72236	70401	71019	15
16	67258	74002	68539	72817	69800	71610	44	46	67901	73413	69172	72216	70422	70998	14
17	67280	73983	68561	72797	69821	71590	43	47	67923	73393	69193	72196	70443	70978	13
18	67301	73963	68582	72777	69842	71569	42	48	67944	73373	69214	72176	70463	70957	12
19	67323	73944	68603	72757	69862	71549	41	49	67965	73353	69235	72156	70484	70937	11
20	67344	73924	68624	72737	69883	71529	40	50	67987	73333	69256	72136	70505	70916	10
21	67366	73904	68645	72717	69904	71508	39	51	68008	73314	69277	72116	70525	70896	9
22	67387	73885	68666	72697	69925	71488	38	52	68029	73294	69298	72095	70546	70875	8
23	67409	73865	68688	72677	69946	71468	37	53	68051	73274	69319	72075	70567	70855	7
24	67430	73846	68709	72657	69966	71447	36	54	68072	73254	69340	72055	70587	70834	6
25	67452	73826	68730	72637	69987	71427	35	55	68093	73234	69361	72035	70608	70813	5
26	67473	73806	68751	72617	70008	71407	34	56	68115	73215	69382	72015	70628	70793	4
27	67495	73787	68772	72597	70029	71386	33	57	68136	73195	69403	71995	70649	70772	3
28	67516	73767	68793	72577	70049	71366	32	58	68157	73175	69424	71974	70670	70752	2
29	67538	73747	68814	72557	70070	71345	31	59	68179	73155	69445	71954	70690	70731	1
								60	68200	73135	69466	71934	70711	70711	0
	N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.		N. cos.	N. sin.	N. cos.	N. sin.	N. cos.	N. sin.	M.
	47 Deg.		46 Deg.		45 Deg.				47 Deg.		46 Deg.		45 Deg.		

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